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Decision making under uncertainties for supply chain optimization

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Decision making under uncertainties for supply chain optimization

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DEDICATION

I would like to dedicate this dissertation to my parents and to my wife without whose support I would not have been able to complete this work.

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ABSTRACT

Efficient supply chain design and operation are essential for manufacturing production. The main stakeholders in a supply chain network include upstream suppliers, downstream customers and competitors. Manufacturing plants acquire raw materials from upstream suppliers and convert them to final products, which are shipped to downstream facilities such as distribution centers and other manufacturing plants. At the strategic level, supply chain management involves designing the configuration of the network, i.e., where to build manufacturing plants, warehouses and distribution centers. At the tactical level, supply chain management involves purchasing raw materials from upstream suppliers, production planning, distribution of products to downstream customers. At the operational level, supply chain management involves demand fulfillment, inventory control and transportation. In this dissertation, mathematical models have been formulated to study various manufacturing supply chain problems, with a focus on decision making under uncertainties for network design, production planning and closed-loop supply chain.

In the first paper, I proposed a novel two-stage stochastic programming model for relay network infrastructure to improve work-life balance of truck drivers. Various valid inequalities have been generated to enhance computational performance and results indicated that up to 30% computational time can be saved. The robustness of the model has been tested by generating scenarios and checking the feasibility of the deterministic model. In addition, we identified the bottlenecks in the system and provided insights on how to improve current network configuration.

In the second paper, I studied a lot-sizing and scheduling problem, which is at the tactical level of supply chain management. Decisions include determining batch sizes and production sequences. A multi-stage stochastic programming model has been developed. Scenario generation and reduction have been used to generate scenarios and identify the most representative subset. A case study based on a manufacturing firm has been conducted to illustrate and verify the model. Results show that by using the multi-stage stochastic programming model, the objective values reduced by 10% - 13% compared to the two-stage stochastic programming model.

In the third paper, I proposed a hybrid stochastic and robust optimization model for the lot-sizing and scheduling problems. Different types of uncertainties (demand and overtime processing cost) have been studied, simultaneously. I assumed there was not enough historical data for demand and hence robust optimization was adopted to handle demand uncertainty. On the other hand, I assumed there was sufficient historical data for overtime processing cost, therefore, stochastic programming was used to handle overtime processing cost uncertainty. Various sensitivity analyses have been conducted and results shown that considering uncertainties are very crucial since the hybrid model outperformed the deterministic model in the objective values.

In the last paper, I studied a closed-loop supply chain problem which integrates network design and production optimization. A fuzzy multi-objective mixed integer linear programming model has been proposed. The two objective functions are minimization of overall system costs and minimization of negative environmental impact. Several uncertain parameters are studied such as demand, return, scrap rate, manufacturing cost and negative environmental factors. The original model with uncertain parameters is firstly converted to a crisp model and then an aggregation

function is applied to combine the objective functions. Sensitivity analyses on various parameters have been examined.

In order to improve the data utilization and interpretation of outcomes, various statistical methods such as Monte-Carlo simulation, moment matching method for scenario generation, and Fast Forward Selection for scenario reduction are applied. The main goal of this dissertation is to quantify the uncertainties in the supply chain design and operational planning processes. Insights have been provided for decision makers in network design and production planning. The results derived from this dissertation have the potential to contribute to the decision making processes under uncertainties by providing analytic solutions for designing robust and efficient supply chain networks.

CHAPTER 1. GENERAL INTRODUCTION

1.1 Research background

A supply chain is a network of organizations that are involved, through upstream and down stream connections, in the different processes that manufacture products delivered to the customers (Christopher et al., 1991). According to a report by the Australian Securities and Investments Commission, 44% of businesses in Australia failed because of poor strategic management. Another survey by Deloitte indicates that 79% of companies with high-performing supply chains achieve revenue growth superior to the average within their industries while only 8% of businesses with low-performing supply chains report above-average growth. These highlight the importance of supply chain design and management.

One of the biggest challenges in the supply chain is the design of network configuration. An inefficient network system results in unnecessary capital investment and huge deadhead cost. Mele (1989) observed that the turnover rate for long haul truck driver ranges from 85% to 110% per year. By contrast, this number is much lower among local drivers. In addition, deadhead costs are significant higher in the traditional point-to-point shipment. It is shown that trucks are completely empty 25% of time and utilization is only 57%. Therefore, designing an efficient network becomes very critical.

Production planning and scheduling has been proven to be one of the most challenging subjects for supply chain management due to its model complexity (Drexl and Kimms, 1997). It appears to be a hierarchical processes ranging from short-term, medium-term to long-term decisions. Our focus is on the medium-term decision making processes which include lot-sizing and scheduling over a finite planning horizon. Lot-sizing and scheduling problems determine the batch sizes as well as production sequences for different products so as to minimize the overall cost and maximize the profit. These specific types of integer programming formulations have been proven to be NP hard and sequence dependent setups will bring even more computational challenges.

Closed-loop supply chain system has attracted increasing attention in the community of supply chain management. It is the design, control, and operation of a system to maximize profit over the entire life cycle of products (Guide Jr and Van Wassenhove, 2009). Unlike traditional supply chain systems, environmental performance is typically considered in the closed-loop supply chain management. Environmentally friendly materials and processes can be incorporated in the supply chain system to realize a low carbon production system (Kumar and Kumar, 2013).

In summary, the four research studies in this dissertation focus on the different components of a supply chain system. In the first paper, I studied a facility location design problem in which a relay network without long distance shipments was provided. A two-stage stochastic programming model with demand uncertainty was proposed. In the second paper, I studied a lot-sizing and scheduling problem with demand uncertainty. A multi-stage stochastic programming model was proposed. First two studies assume perfect information about uncertainty which can be unrealistic in some cases. In the third paper, we proposed a hybrid stochastic and robust optimization model for the lot-sizing and scheduling problems in which uncertain demand is handled by the robust optimization due to lack of historical data. The last paper investigated a closed-loop supply chain problem in which returned products are collected and remanufactured. A fuzzy multi-objective programming model was proposed to handle various uncertainties such as demand, return, scrap rate, manufacturing cost and negative environmental impacts.

1.2 Literature review

An efficient and robust supply chain network can contribute to the competitiveness of a manufacturing organization by reducing costs and mitigating risks. As pointed out by Klibi et al. (2010), the configuration of a supply chain network is a critical decision, which will affect multiple departments and functionalities, especially under uncertainties. On top of that, network planning is a long term process in which decisions such as opening facility can be very time and resource consuming. Therefore, the design of supply chain should be robust against uncertainties. In this dissertation, I studied three topics in the scope of supply chain design (1) relay network design; (2) lot-sizing and scheduling; (3) closed-loop supply chain. Different methods have been applied for model formulation and analysis, including stochastic programming, robust optimization, and fuzzy programming.

In the traditional supply chain network, point-to-point shipment is widely used regardless of the distance between two nodes. This traditional transportation approach creates huge amount of deadhead cost and high turnover rate. Unlike a traditional network, a relay network connects source and sink with intermediate nodes whenever the distance between nodes is greater than the transmission range. Üster and Kewcharoenwong (2011) designed a relay network system in truckload transportation that potentially helped to alleviate the problem of high turnover rate. An efficient Benders' decomposition-based algorithm was developed. Kulturel-Konak and Konak (2008) studied a network design problem with relay stations arises in telecommunication. An efficient hybrid metaheuristic approach was presented to solve large sized problems. Other approaches for solving relay network design problems include benders decomposition, genetic algorithm, lagrangian heuristic and branch-and-price (Konak, 2014; Yıldız et al., 2018; Kewcharoenwong and Üster, 2014). While most of the current literature focused on developing heuristics, I proposed a novel mathematical formulation for relay network design problems and improved the computational performance by introducing various valid inequalities. Stochastic programming was applied to handle demand uncertainty. A two-stage stochastic programming model was formulated to minimize transportation cost, deadhead cost, penalty cost, and fixed cost. A case study on the highway network for the Western United States demonstrated the computational tractability of the approach along with the importance of considering demand uncertainty (Hu et al., 2019).

Another important component in supply chain design is production. Manufacturing firms can experience various uncertainties both externally and internally. External uncertainties include lead time, raw material quality, demand, etc. Internal uncertainties include efficiency, machines break down and processing cost, etc. Lot-sizing and scheduling is a type of production problems in which batch sizes and production sequences need to be determined. Two-stage stochastic programming approaches have been widely studied for lot-sizing and scheduling problems (Hu and Hu, 2016; Ramaraj et al., 2017; Li and Hu, 2017). The major motivation to use multi-stage stochastic programming approach for lot-sizing and scheduling problems is that decision makers are usually able to revise production plan at the beginning of each time period and make changes accordingly. In the second paper, a multi-stage stochastic programming approach was proposed to address the uncertain demand issue in a production scheduling problem. The major connection between the first and second papers, in terms of methodology, is that both papers adopt stochastic programming method. As mentioned before, opening facilities can be very time-consuming and the effects of facility location decisions are long-lasting. In addition, facility location decisions cannot be revised frequently, therefore, two-stage stochastic programming is more suitable for network design problem. Conversely, production including regular and overtime production plans can be revised on the weekly or monthly basis depending on the firm's policy and hence multi-stage stochastic programming is more suitable for lot-sizing and scheduling problems.

Scenario based stochastic programming is a powerful modeling approach if an accurate probability distribution of the random variable is known. However, data source may not be sufficient to generate distributions due to incompleteness or unavailability of data. On the other hand, even there is sufficient data, a representative approximation may require a large number of scenarios which increases computational complexity significantly. Conversely, if the scenario sample size is restricted due to computational limitations, the possible outcome of future stages under which decisions are determined and evaluated is limited. In addition, scenario based stochastic programming focuses on the average performance of the system while there are situations where the decision maker concerns more about the worst case result. Bertsimas et al. (2018) proposed a robust optimization approach based on polyhedral uncertainty sets. The advantage of their approach is that the counterpart of a linear programming problem remains a linear programming problem. Literature on robust optimization for lot-sizing and scheduling problems with applications of sawmill and refrigerator production system can be found in (Varas et al., 2014; Rahmani et al., 2013; Kanyalkar and Adil, 2010). The major contribution of our third paper is that I apply a hybrid approach for lot-sizing and scheduling problems. Two types of uncertainties were studied in the paper: demand and overtime processing cost. I assumed that there was enough historical data for overtime processing cost and hence stochastic programming was adopted to handle overtime processing cost uncertainty. However, estimation of the distribution for future demand may not possible due to market complexity and as a result, robust optimization was adopted to deal with demand uncertainty. Similar approach has been studied in other applications such as supply chain network design and power system markets (Keyvanshokooh et al., 2016; Fanzeres et al., 2015). The major connection between the second and third paper is that I employ different approaches based on the modeling assumption. If there is sufficient historical data and computation power, then scenario based stochastic programming should be used. Reversely, if there is limited amount of data and decision maker concerns more about the worst-case performance, then robust optimization should be used.

Social, environmental and economic concerns spur an interest to develop sufficient and robust supply chain networks. Kumar et al. (2004) applied fuzzy programming approach for solving vendor selection problem. Three objectives include minimizing net cost, net rejections and net late deliveries. Pishvaee and Torabi (2010) studied a closed-loop supply chain problem using fuzzy programming. Minimizing overall costs and total delivery tardiness were considered in the model. Since most supply chain problems may have multiple objectives, various aggregation functions have been studied for fuzzy programming such as weighted additive and min-max approaches (Amid et al., 2011, 2009). To address the sub-optimality problem from separated design in forward and backorder networks, a multi-objective closed-loop supply chain problem is studied in my fourth paper. The goal is to minimize overall system cost while maximizing the utilization of environmental friendly materials. In terms of methodology, I adopted fuzzy programming since supply chain network design is a long term planning problem and hence most parameters can be uncertain. Scenario based stochastic programming requires accurate joint distribution of all uncertain parameters which are often impossible to obtain. In addition, closed-loop supply chain problems with large scenario sample size is usually prohibitive due to computational limitations. According to Bertsimas et al. (2018), only the coefficients on the left are affected by uncertainties. If both left-hand and right-hand sides are affected, simultaneously, then auxiliary variables are required which increase the complexity of problems. The fuzzy programming approach is more suitable for closed-loop supply chain problems since the complexity of this approach is independent of number of uncertain parameters. Another connection between the third and fourth papers that differs from previous papers is that robust optimization and fuzzy programming approaches capture the decision maker's attitude towards risk. A large budget in the robust optimization provides conservative solution and the probability of constraint violation is low. In the fuzzy programming approach, α indicates the degree of feasibility. Lower α value leads to better objective value at the expense of lower degree of feasibility. Obviously, the robust optimization and fuzzy programming approaches take solution feasibility into consideration while scenario based stochastic programming approach does not.

1.3 Dissertation structure

The remainder of the dissertation is organized as follows. The first paper on relay network design for daily routes is presented in chapter 2 and has been published in the International Journal of Production Research. In chapter 3, I present a multi-stage stochastic programming model for lot-sizing and scheduling problems and this paper has been published in the Computers and Industrial Engineering. In chapter 4, I propose a hybrid stochastic and robust optimization approach for lot-sizing and scheduling problems. This paper is currently under second round revision in the European Journal of Operational Research. In chapter 5, I present a multi-objective fuzzy programming approach for closed-loop supply chain problems which has been submitted to the Computers and Industrial Engineering. Finally, chapter 6 includes conclusions, limitations, and future works for this dissertation.

CHAPTER 2. HUB RELAY NETWORK DESIGN FOR DAILY DRIVER ROUTES

Hub-based relay networks for long haul trucking offer an opportunity to improve the work-life balance of drivers while simultaneously supporting faster delivery through near-continuous flow of containers from source to destination. In this paper, we develop a model for deciding hub location and sizing along with the routing of loads. Costs of hub construction and operation, transportation and penalties for multi-day driver trips are included. Both deterministic and two-stage stochastic programming models have been formulated in this paper. The goal is to determine the optimal hub and route decisions so that overall cost is minimized. A case study on the highway network for the Western United States demonstrates the computational tractability of the approach along with the importance of considering demand uncertainty.

2.1 Introduction

Long haul trucking is a major component of the U.S. logistics system. The Bureau of Labor Statistics reported that there were approximately 1.8M long haul truckers in the U.S. in 2014. The U.S. Department of Transportation data indicate that over 60% of total freight weight and volume are shipped by truck, accounting for over $12B annually. These truck shipments account for over 40% of the ton-miles of shipped freight. The majority of these shipments exceed 250 miles and one tenth are for shipments in excess of 2,000 miles. Given the extent of this activity, system efficiency and sustainability are of critical importance. Efficiency refers to system costs and service (delivery lead time and reliability). Sustainability refers to both operational and environmental sustainability.

Truck shipments are typically classified as full truckload trucking (TL) or less than truckload trucking (LTL). While trucks can often carry 20,000 lbs or more of cargo, generally loads in excess of 10,000 lbs are considered TL. Large suppliers may make direct Point-to-Point (PtP) shipments to major customers when demand volume and desired delivery frequency warrant direct TL shipments. However, in many instances smaller loads are consolidated into full TL loads at local hubs to facilitate cost efficiency. Shipments are rarely symmetric and TL research has focused on how to minimize the unloaded (deadhead) movement of empty trucks returning to their home base or next pick up location. Even though long PtP routes require multiple days per trip, to reduce deadheading routes are often created with multiple legs causing drivers to be away from home for extended periods. This method has substantial benefits for companies and customers in terms of cost, but it is unsustainable both mentally and physically for truck drivers. Long driving distance increases driver turnover rate. Mele (1989) observed that turnover ranges from 85% to 110% per year in the TL industry. That problem persists today creating a shortage of long haul truck drivers. By contrast, the turnover rate for local drivers with daily tours is significantly lower. Safety is also an issue as drivers who are assigned away from home for a long time incur diet and sleep problems that can increase the accident rate. In addition, the deadhead issue impacts costs. Meller et al. (2012) reported that trucks are completely empty 25% of total time and utilization is only 57% for the other 75% of loaded time.

The Federal Motor Carrier Safety Administration maintains Hours of Service Regulations for truck drivers. Included are an 11-Hour Driving limit and a 14-Hour consecutive duty limit following 10 consecutive hours off duty. Thus, by regulation, a lone driver transporting a load can be actively advancing that load at most 50% of the time. This creates the possibility for designing a logistics system that is more amenable to driver work-life balance, safety and delivery speed. This forms the motivation for this research. Our objective is to develop models to create a hub relay network either for a single large carrier, a consortium of carriers or as a third party enterprise. The models are intended to determine the capacity and location of relay hubs along a highway network to facilitate primarily, if not solely, single day driver tours. We assume a finite set of feasible hub locations and sizes are available. A set of daily flows of freight loads between sources and destinations is assumed to be known either as deterministic parameters or with a distribution. Costs considered include the period depreciation and operating cost of a hub by size and location, loaded and deadheading travel costs per mile for each highway link and a penalty cost per mile or time for overnight trips. Accompanying the hub location decision is the route to be taken for each load.

The major contributions can be summarized as follows:

- We develop a novel volume and link-length capacitated, hub design model as a mixed-integer linear program. Deadhead cost and extra travel time are measured;
- A time-dependent penalty cost is included in the objective function for multi-day tours. A higher penalty cost rate parameter results in larger fixed cost for facility construction and drivers return home more frequently. A lower penalty cost rate leads to longer round trips but fewer hubs are required;
- We extend the deterministic model to a two-stage stochastic programming model by considering uncertainty in demand. Robustness of, and bottlenecks in, the stochastic system are examined;
- Various preprocessing cuts and two sets of feasibility cuts are developed to enhance computational performance;

The reminder of the paper is organized as follows: section 2.2 reviews the relevant literature on hub networks applicable to trucking. Deterministic and stochastic demand models are formulated in section 2.3. Computational results and sensitivity analyses are provided in section 2.4 followed by Conclusions in section 2.5.

2.2 Literature review

Hub and spoke networks have been used in the airline industry for forty years (Phillips, 1987). However, in the airline industry, the goal is to consolidate passengers and take advantage of the multiple aircraft sizes, which differs from that for the hub network considered for trucking. Focusing on LTL, Braklow et al. (1992) described freight transportation practices and the use of hubs for consolidation. While opportunities for load consolidation and cross-docking exist and would be supported by the proposed hub network, our primary motivation is to support one-day tours for drivers and continuous movement of loads. The concept of using such a hub network for trucking is not new. Taylor et al. (1999) considered the problem from three different viewpoints: i) customer; ii) company; and iii) driver. From a customer's perspective, good dispatching means merchandise can be delivered on time without damage. From a company's perspective, a good dispatching scenario means high equipment utilization and good delivery performance. From a driver's perspective, a good hub dispatching scenario can be well represented by the total mileage per day, which is directly proportional to pay.

A hub network is consistent with the concept of the Physical Internet (PI), an initiative to rethink logistics making use of standardized shipping containers and coordinated, cooperative shipment (Montreuil et al., 2013). Meller et al. (2012) compared the conventional dispatching model with a PI hub model. Under conventional dispatching, a single driver picks up loads at source point and delivers them to the destination. For PI dispatching, drivers pick up freight originating in their domicile zone, drop off loads at zone boundary or hubs and pick up incoming loads for delivery within their own zone. Several advantages of this new model would include: (i) drivers could still use conventional dispatching methods to maintain many of the efficiencies inherent to the conventional dispatching system, (ii) drivers could always travel only in their own domiciles, so that they can go back home more frequently, (iii) drivers are more familiar with driving conditions and routes, and (iv) stress and fatigue are reduced potentially resulting in fewer accidents. However, Taylor et al. (2001) mentioned that for loads with short haul lengths the performance of PI could become poor. Finally, while not directly part of this paper, we note that a hub relay network could assist with recent innovations in city logistics. Proper placement of hubs and scheduling of load handoffs could facilitate inner city deliveries at off-peak hours thereby reducing congestion and improving environmental sustainability.

Most of the prior hub design literature has focused on problems with a limited number of links per trip and limited outgoing/incoming links from sources/destinations. MacKinnon and Barber (1972) introduced a network problem of designing transportation system including of truck lines. Qiu and Sharkey (2013) considered a dynamic single facility location problem of building a sea base, the goal was to minimize total transportation cost as well as to provide necessary supplies to military locations. O'Kelly (1986) modeled the cases where all flows must go through a single hub or at most two. The two hub problem implies at most one interhub link per source to destination route. Unlike these facility location problems, several new facilities must be located with respect to existing hubs in the PI network design problem. Campbell (1996) modeled the problem as a p-hub Median problem assuming at most two hubs would be used between any source-destination pair and developed heuristics for the single allocation per source/destination case. Motivated by postal delivery, Ebery et al. (2000) examined several formulations for the problem with at most two hubs per trip and proposed an heuristic. Hub capacities were considered. Üster and Agrahari (2011) also modeled the problem with at most two intermediate stops, essentially a consolidation and deconsoidation point. Campbell and O'Kelly (2012) provided a more recent survey of this research area. In our problem, however, each source and destination can interact with multiple hubs and the number of links on a path is unlimited (except cycles are not allowed) to allow for shorter driver routes. Üster and Maheshwari (2007) considered a strategic network design for multi-zone truckload shipments. They formulated a model in order to choose the location of each Relay Node (RPs are equivalent to our hubs), service area of each RP and assignment of non-RP nodes to RPs. Üster and Kewcharoenwong (2011) modified the model and developed a relay network for truckload transportation. The relay network problem addressed in those papers is very similar to the problem addressed in this paper. Efroymson and Ray (1966) proposed a discrete plant location problem and the purpose was to determine quantity, location, and size of plants. The differences are: (i) We do not restrict paths to be unique thus allowing continuous flow variables; (ii) We impose capacity limits on hubs based on fixed costs with discrete capacity choices; (iii) We do not restrict the number of links on a path as in Uster and Agrahari; (iv) We directly model the costs of deadheading and extended tours instead of utilizing hard constraints; and (v) we focus on the interhub traffic, assuming all regional transit is handled separately and therefore effectively originates or terminates at a major hub location. In practice this would mean a major port or urban area.

There are also papers focusing on developing heuristics to solve network design problems. Cooper (1964) developed an heuristic and pointed out that a random destination algorithm provided the closest optimal result for the allocation problem in a reasonable amount of computation time. Kuehn and Hamburger (1963) developed a greedy algorithm by adding one warehouse a time until no additional warehouse can be built without increasing total cost. Feldman et al. (1966) introduced an opposite heuristic algorithm by removing one existing warehouse a time. They claimed that no solutions generated were higher in cost than the Kuehn and Hamburger's results, but no big improvements were observed either. Love (1974) constructed a multi-facilities location problem using dual program as it solved location problems much more efficient in the special cases where linear constraints are presented. Our models are shown to be computationally feasible for mid-sized problems, so we decide to reserve the development of new algorithm for our future works.

2.3 Problem definition and formulation

The operational characteristic of a PI-inspired hub network is shown in Figure 2.1. The suppliers and customers are represented by circles and the hubs are represented by squares. The maximum allowable travel time forces drivers to drop off freight at a site where a hub is built. Route efficiency, capacity and fixed cost are critical factors that make one hub location more preferable than others. In Figure 2.1, one feasible route to ship products from supplier i to customer l is to stop at four different hubs before reaching the destination. The dashed horizontal line between nodes i and l is the conventional delivery route. The motivations to design a PI network instead of delivering products from point to point can be summarized as follows:

- By assigning drivers to particular work areas, they will be more familiar with the routes and this will reduce the chance of an accident;
- Asymmetric flow volumes result in point to point delivery usually incurring huge deadhead cost. This issue can be addressed in the hub network system by allowing drivers to have flexible routing options;
- Drivers can return home nightly or at least much more often in the hub network system than in the conventional network system thus increasing driver satisfaction and reducing turnover and travel cost;
- The potential exists to exchange loads for a more continuous flow without necessarily incurring the idle time due to driver rest periods.

In modeling the hub network, we make several assumptions as follows:

- The set of shipments (source-sink pairs and load volume per time) is known either deterministically or through a probability distribution;

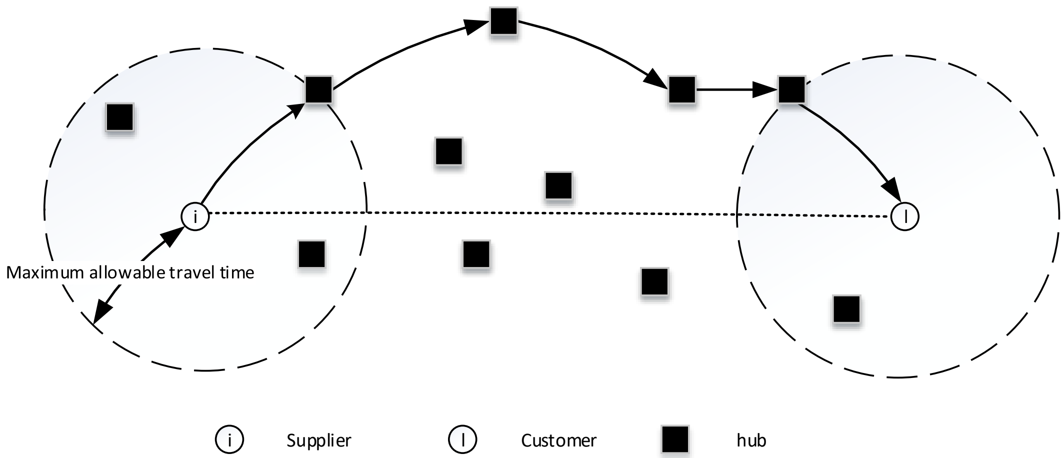


Figure 2.1: A simplified example of PI-inspired network

- As the intent is to facilitate single day tours of no more than 10 hours, an hourly overtime penalty is charged for trips that do not permit the driver to return to their domicile or home base in a day. Corresponding policy on Hours-of-Service limits can be found on (https://www.foleyservices.com/news/hours-of-service-the-100150-air-mile-exemptions/comment-page-8/);
- To minimize long hauls, an upper limit is set on the length of an outbound and return tour for a driver. Longer segments and tours are forbidden;
- A hub with sufficient capacity is needed if one wants to send out flow from a source node or receive flow at a sink node. This assumption can be found in the most production problems in which warehouses are used for storage;
- Hub capacity is measured by the maximum of the incoming or outgoing loads per period;
- A potential hub location (typically a city) can have multiple distribution centers as dictated by capacity requirements. For example, there are multiple FedEX warehouses in Phoenix, AZ;
- Travel time can be asymmetric, that is, the travel time from point A to point B may be different from the travel time from B to A;
- The model implicitly assumes an operational ability to manage flows along arcs such that deadheading will occur in at most one direction on each arc. This is the intention of the drop-off and pickup nature of the PI-inspired hub network.

2.3.1 Parameters and decision variables

The mathematical notation is described in Table 2.1. Let $N = \{1, \cdots, n\}$ represent the set of potential hub locations. $I = \{1, \cdots, i\}$ denotes the set of suppliers (shipment sources), $L = \{1, \cdots, l\}$ indicates the customers, and $Z = \{1, \cdots, z\}$ indexes the discrete choices of hub size. Note that I and L are subsets of N since N includes all source and sink nodes. Parameter w_{il} is the demand from site i to site l and t_{jk} denotes the travel time between adjacent site locations j and k. In addition, p_{jk} is the penalized travel time if we directly go from j to k. For example, if the allowable daily tour is 10 hours and $t_{jk} = 8$ then $p_{jk} = 8 - 5 = 3$ since there is no penalty for the first 5 hours of outbound travel time but longer segments imply a tour exceeding one day. The regular transportation cost per unit travel time is α and β is the penalty cost for working overtime. Let U_{jz} and c_{jz} indicate the capacity in loads per period and the fixed depreciation and operational cost of hub of type z at location j, respectively. Deadhead cost per unit of travel time is measured by the parameter δ and presumably $\delta \leq \alpha$. Decision variable X_{ijkl} represents the proportional demand flow from i to l using j and k as adjacent stops. There is no limit on the number of hubs visited for any flow. X_{iikl} denotes the first segment of shipment going from supplier i to hub k. X_{ijll} denotes the last segment of shipment going from hub j to customer l. As shown in Figure 2.1, the feasible route has two more hubs to visit. Note that X_{ijkl} represents the flow from supplier i to customer l using adjacent stops j and k. Unlike the p-hub problem with fixed p value, our problem allows drivers to visit any number of hubs in the optimal solution. Y_{jz} is a binary variable which takes value one if a hub of type z is built at site j and zero otherwise. Finally, H_{jk} measures the flow imbalance on the arc $\{j, k\}$, i.e. deadheading. From a mathematical perspective, $H_{jk} = \left| \sum_{i=1}^{I} \sum_{l=1}^{L} w_{il} \cdot (X_{ijkl} - X_{ikjl}) \right| \forall \{j, k\} \in N$. This nonlinear function can be linearized by adding two extra constraints in the model : $\sum_{i=1}^{I} \sum_{l=1}^{L} w_{il} \cdot (X_{ijkl} - X_{ikjl}) \leq H_{jk}$ and $\sum_{i=1}^{I} \sum_{l=1}^{L} w_{il} \cdot (X_{ikjl} - X_{ijkl}) \leq H_{jk}$

2.3.2 Capacitated, deterministic hub design model

In the deterministic model, volume of demand is fixed and known. We aggregate daily demand and analyze the system based on the annual demand. Later in the stochastic programming model, we use daily demand to allow each supplier-customer pair to have different demand on a daily basis. Let F be the set of source-destination pairs for which $w_{il} > 0$. The CApacitated, Deterministic HUb network design model (CADHUB) can be shown as follows:

$$\begin{aligned} \min \sum_{i=1}^{I} \sum_{i \neq l}^{} \sum_{l=1}^{L} \sum_{j=1}^{N} \sum_{k=1}^{N} w_{il} \cdot X_{ijkl} \cdot (t_{jk} \cdot \alpha + p_{jk} \cdot \beta) \\ + \sum_{j=1}^{N} \sum_{k>j}^{N} \delta \cdot H_{jk} \cdot t_{jk} + \sum_{j=1}^{N} \sum_{z=1}^{Z} C_{jz} \cdot Y_{jz} \end{aligned} \tag{2.1}$$

subject to:

$$\sum_{k=1}^{N} X_{iikl} - \sum_{k=1}^{N} X_{ikil} = 1 \qquad \forall \{i, l\} \in F \tag{2.2}$$

Table 2.1: Notation for the mathematical model

Indices		
i	$1, 2 \cdots I$	Suppliers
l	$1, 2 \cdots L$	Customers
j, k	$1, 2 \cdots N$	Potential hub locations
z	$1, 2 \cdots Z$	Capacity levels
Parameters		
w_{il}	Demand from i to l	
t_{jk}	Travel time from j to k	
p_{jk}	Overtime from j to k	
U_{jz}	Capacity level z	
C_{jz}	Fixed cost of a hub at capacity level z	
α	Transportation cost per hour	
β	Penalty cost for overtime per hour	
δ	deadhead cost per hour	
Decision Variables		
X_{ijkl}	Proportional demand from i to l using hubs j and k as adjacent stops	
Y_{jz}	Binary variable takes 1 if a hub at capacity level z is built at site j and 0 otherwise	
H_{jk}	Total amount of deadhead from j to k	

$$\sum_{j=1}^{N} X_{ijll} - \sum_{j=1}^{N} X_{iljl} = 1 \qquad \forall \{i, l\} \in F \tag{2.3}$$

$$\sum_{j=1}^{N} X_{ijkl} - \sum_{j=1}^{N} X_{ikjl} = 0 \qquad \forall \{i, l\} \in F, k \neq i, k \neq l \tag{2.4}$$

$$\sum_{i=1}^{I} \sum_{i \neq l \; l=1}^{L} \sum_{k=1}^{N} w_{il} \cdot X_{ijkl} \leq \sum_{z=1}^{Z} U_{jz} \cdot Y_{jz} \qquad \forall j \tag{2.5}$$

$$\sum_{i=1}^{I} \sum_{i \neq l \; l=1}^{L} \sum_{j=1}^{N} w_{il} \cdot X_{ijkl} \leq \sum_{z=1}^{Z} U_{kz} \cdot Y_{kz} \qquad \forall k \tag{2.6}$$

$$\sum_{i=1}^{I} \sum_{i \neq l \; l=1}^{L} w_{il} \cdot (X_{ijkl} - X_{ikjl}) - H_{jk} \leq 0 \qquad \forall \{j, k\} \tag{2.7}$$

$$\sum_{i=1}^{I} \sum_{i \neq l \; l=1}^{L} w_{il} \cdot (X_{ikjl} - X_{ijkl}) - H_{jk} \leq 0 \qquad \forall \{j, k\} \tag{2.8}$$

$$0 \leq X_{ijkl} \leq 1 \qquad \forall\{i,j,k,l\} \qquad H_{jk} \geq 0 \qquad \forall\{j,k\} \tag{2.9}$$

$$Y_{jz} \in \{0,1\} \qquad \forall\{j,z\} \tag{2.10}$$

The first term in the objective function is the overall transportation costs between all of the source and sink nodes. This term accumulates both the regular time and any additional time-based penalty cost. The second term represents the total deadhead cost in the system. The cost is a lower bound based on the minimum imbalance in loaded and unloaded flows along each arc. The third term is the total fixed depreciation and capacity-based operational cost for building hubs. Maximum allowable trip travel time θ is implicitly restricted by setting travel time to a relatively large constant if its real travel time is greater than θ. Alternatively assign a relatively large penalty cost or zero flow for those travel times larger than θ. Equation 2.2 restricts that flow will never come back to the source after being sent out. Equation 2.3 ensures that flows terminate at the intended destination. Equation 2.4 is a flow conservation constraint. It ensures that flow coming out from a site equals to flow going into that site except the source node and sink node. Equation 2.5 and Equation 2.6 restrict flow into and out of a location to be no more than the capacity of the hub constructed at that site including the source node and sink node. Equation 2.7 and 2.8 evaluate the deadhead cost on arc $\{j,k\}$. H_{jk} is set to the maximum of the total weighted loaded flow differential in either direction between j and k. Equation 2.9 includes the non-negativity and the upper bound on X and H variables. Equation 2.10 restricts Y variables to be binary. In reality, it is possible to have multiple distribution centers or storages at one site and this is considered in our model as well. The fixed cost of building a hub is typically a concave function indicating the hub with larger capacity should have lower unit cost. In addition, four different types of preprocessing constraints are implemented: (i) No flow on an arc if its travel time is longer than the maximum travel time θ; (ii) No flow reentering its source, which can be expressed as $X_{ijil} = 0$; (iii) No flow leaving its sink, which can be expressed as $X_{ilkl} = 0$; (iv) Flow directly from sink to source is not allowed, which can be expressed as $X_{ilil} = 0$.

An advantage of the model is the use of continuous variables to represent flows. Only hub location and sizing decisions require binary variables. This formulation has $O(MN^2)$ continuous variables and $O(NZ)$ binary variables.

The model is also flexible and can be extended to consider other factors. For example, some states allow double or triple loads. This can affect both transportation cost and deadhead cost. The hub model could account for such scenarios if they were predetermined along specific legs of source-destination pairs by defining parameters α_{ijkl} and δ_{jk} appropriately. Likewise, the number of paths to be considered for each source-destination pair could be restricted to likely, pre-identified best choices. This would reduce the number of variables and strengthen the feasibility cuts generated based on hub capacity. Note that we have chosen to model costs in terms of trip time but the objective may be easily modified to reflect costs as a function of mileage or a combination of time and distance.

2.3.2.1 Two-stage stochastic programming model

The CADHUB model assumes flows are known. In practice, only forecasts are available and decisions are often made under uncertainty. In this section, demand w_{il} is considered as a random variable whose realization is known only after the hub locations and sizes are specified. Daily demands vary according to a probability distribution. A two-stage stochastic programming model is proposed to assist the decision making under uncertainty. Decision variables can be classified chronologically into two categories: first-stage decisions and second-stage decisions. The first-stage decisions have to be determined in the presence of uncertainty while the second-stage decisions can be made after realization of shipment demand. In the strategic network design problem, the first-stage decisions include determination of hub locations and capacities. The second-stage decisions determine the route based on the existing hub locations, hub capacity, and the realization of uncertain demand. In the case of period to period variability, we assume the system essentially clears itself each period and ignore potential inventory accumulations between periods. Thus, hub location Y_{jz} is first determined in the presence of uncertain demand. Then, routing decision X_{ijkl} is made for the realized shipments given those hub locations and capacities.

Let the flow volume demand be represented by the $n(n-1)$ component random vector $\xi = \{w_{1,2}, ..., w_{l,l-1}\}$ where w_{ij} is the number of truck loads to be transported in the period from i to j. Note that flows may be in either direction. The objective is to minimize the hub configuration and expected transport cost subject to Equation 2.10:

$$\min \sum_{j=1}^{N} \sum_{z=1}^{Z} C_{jz} \cdot Y_{jz} + E[O(X, H, \xi)] \tag{2.11}$$

$E[O(X, H, \xi]$ is the expected value of the routing problem for a given hub network Y_{jz} and flow volume scenario ξ_s defined by:

$$\min \sum_{i=1}^{I} \sum_{i \neq l \, l=1}^{L} \sum_{j=1}^{N} \sum_{k=1}^{N} w_{il}(\xi_s) \cdot X_{ijkl} \cdot (t_{jk} \cdot \alpha + p_{jk} \cdot \beta) + \sum_{j=1}^{N} \sum_{k>j}^{N} \delta \cdot H_{jk} \cdot t_{jk} \tag{2.12}$$

s.t.

$$\sum_{i=1}^{I} \sum_{i \neq l \, l=1}^{L} \sum_{k=1}^{N} w_{il}(\xi_s) \cdot X_{ijkl} \leq \sum_{z=1}^{Z} U_{jz} \cdot Y_{jz} \qquad \forall \{j\} \tag{2.13}$$

$$\sum_{i=1}^{I} \sum_{i \neq l \, l=1}^{L} \sum_{j=1}^{N} w_{il}(\xi_s) \cdot X_{ijkl} \leq \sum_{z=1}^{Z} U_{kz} \cdot Y_{kz} \qquad \forall \{k\} \tag{2.14}$$

$$\sum_{i=1}^{I} \sum_{i \neq l \, l=1}^{L} w_{il}(\xi_s) \cdot (X_{ijkl} - X_{ikjl}) - H_{jk} \leq 0 \qquad \forall \{j, k\} \tag{2.15}$$

$$\sum_{i=1}^{I} \sum_{i \neq l \, l=1}^{L} w_{il}(\xi_s) \cdot (X_{ikjl} - X_{ijkl}) - H_{jk} \leq 0 \qquad \forall \{j, k\} \tag{2.16}$$

In addition, Equation 2.2, 2.3, 2.4, 2.9 are also required to minimize Equation 2.12.

The demand is assumed to be normally distributed that can be approximated by a set of possible scenarios where subscript s is used to denote a scenario with probability P_s. Each w_{ils} represents a possible realization of uncertain daily demand and there are 10 different patterns (Hu and Hu, 2016). Letting the set of scenarios approximate the volume of demand, we obtain the deterministic equivalent CAPHUB model

$$\begin{aligned}\min \sum_{s=1}^{S} P_s(\sum_{i=1}^{I} \sum_{i\neq l} \sum_{l=1}^{L} \sum_{j=1}^{N} \sum_{k=1}^{N} w_{ils} \cdot X_{ijkls} \cdot (t_{jk} \cdot \alpha + p_{jk} \cdot \beta) \\ + \sum_{j=1}^{N} \sum_{k>j}^{N} \delta \cdot H_{jks} \cdot t_{jk}) + \sum_{j=1}^{N} \sum_{z=1}^{Z} C_{jz} \cdot Y_{jz}\end{aligned} \tag{2.17}$$

Subject to:

$$\sum_{k=1}^{N} X_{iikls} - \sum_{k=1}^{N} X_{ikils} = 1 \qquad \forall\{i, l, s\} \tag{2.18}$$

$$\sum_{j=1}^{N} X_{ijlls} - \sum_{j=1}^{N} X_{iljls} = 1 \qquad \forall\{i, l, s\} \tag{2.19}$$

$$\sum_{j=1}^{N} X_{ijkls} - \sum_{j=1}^{N} X_{ikjls} = 0 \qquad \forall\{i, l, k, s\} \tag{2.20}$$

$$\sum_{i=1}^{I} \sum_{i\neq l} \sum_{l=1}^{L} \sum_{k=1}^{N} w_{ils} \cdot X_{ijkls} \leq \sum_{z=1}^{Z} U_{jz} \cdot Y_{jz} \qquad \forall\{j, s\} \tag{2.21}$$

$$\sum_{i=1}^{I} \sum_{i\neq l} \sum_{l=1}^{L} \sum_{j=1}^{N} w_{ils} \cdot X_{ijkls} \leq \sum_{z=1}^{Z} U_{kz} \cdot Y_{kz} \qquad \forall\{k, s\} \tag{2.22}$$

$$\sum_{i=1}^{I} \sum_{i\neq l} \sum_{l=1}^{L} w_{ils} \cdot (X_{ijkls} - X_{ikjls}) - H_{jks} \leq 0 \qquad \forall\{j, k, s\} \tag{2.23}$$

$$\sum_{i=1}^{I} \sum_{i\neq l} \sum_{l=1}^{L} w_{ils} \cdot (X_{ikjls} - X_{ijkls}) - H_{jks} \leq 0 \qquad \forall\{j, k, s\} \tag{2.24}$$

$$0 \leq X_{ijkls} \leq 1 \quad \forall\{i, j, k, l, s\} \qquad H_{jks} \geq 0 \quad \forall\{j, k, s\} \qquad Y_{jz} \in \{0, 1\} \tag{2.25}$$

2.4 Case study

We apply the PI-inspired hub network design framework for a case study in the western United States to illustrate and validate the optimization model. Many of the busiest U.S. ports are mainly located along the west coast with Los Angeles and Long Beach being the busiest two. Together they move on the order of 14,000,000 TEUs (twenty-foot equivalent units, a standard intermodal shipping container) annually. Seattle and Oakland are also in the top 10 busiest ports which move 4,000,000 TEUs combined. Thirty-seven potential hub locations in the West Coast highway network are selected including those four ports. Transportation cost is $ 22 per hour since it is average hourly pay for truckers. Penalty for driving overtime and deadhead cost are set proportional to the regular transportation cost. Demand is obtained from Freight Analysis Framework Data Tabulation (http://faf.ornl.gov/faf4/Extraction2.aspx), and three different sizes of hubs are considered at potential locations. The capacities of hubs are proportional to the overall demand and larger hub has smaller unit cost. The GAMS (General Algebraic Modeling System) is utilized to solve both deterministic and two-stage stochastic programming models. It is a high-level modeling system for the optimization industry which connects to several third party solvers such as CPLEX and Gurobi.

2.4.1 Results of deterministic model

We assume that the trucks used in the system are semi-trailer trucks and the average load per trip is 10 tons. θ is set to 10 hours so each driver will go back home in at most two days. Travel times are taken from (http://www.driving-distances.com/) and flow volumes from the U.S. DOT Bureau of Transportation Statistics.

In this deterministic model, we selected ten cities as our suppliers/customers. Each selected location will send flow to all other selected locations resulting in total 90 flows. Deadhead cost is \$ 2.04×10^8, transportation cost is \$ 1.17×10^9, and hub building cost is \$ 1.42×10^9. Note that these costs are the annual costs since the demand in the deterministic model is fixed, therefore, we aggregate daily demand. Eight small hubs, two medium hubs, and two large hubs are built, respectively. The potential hub locations can be found in Figure 2.2. Comparing to direct shipments, the overall extra travel time is measured by:

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\{w_{il} \cdot (\sum_{j=1}^{N}\sum_{k=1}^{N} X_{ijkl} \cdot t_{jk} - t_{il})\}$$

for any positive demand flow w_{il}. In this expression $\sum_{j=1}^{N} \cdot \sum_{k=1}^{N} X_{ijkl} \cdot t_{jk}$ measures the overall transportation travel time in the PI network and t_{il} measures the travel time of conventional point to point shipment. The extra travel time is 4.93×10^7 hours. The partial results of flows are shown in Table 2.2. We picked three different flow results: (i) flow uses multiple routes and hub stops (Seattle-San Francisco flows); (ii) flow uses single route but multiple different hub stops (Sacramento-Phoenix flows); (iii) flow directly from source to sink (Phoenix-Las Vegas flows). For example, flows from San Francisco (22) to Seattle (1) are represented by $22 \rightarrow 1$. First, all products will be transported from 22 to 25. Then 11.9% of products will be directly delivered to destination Seattle while 88.1% of products will be delivered to 27 followed by destination. Note that bi-directional route decisions can differ between location pairs due to demand imbalance and hub capacities. The travel time between Seattle and San Francisco is around 12.5 hours and the travel time between Sacramento and Phoenix is around 11 hours. The travel times between those cities exceeds the maximum allowable travel time meaning we need at least one stop (drop-off) before arriving at the destinations. The travel time between Phoenix and Las Vegas is less than 5 hours indicating this is a PtP conventional shipment. Since the maximum allowable travel time θ, fixed cost, and penalty cost for overtime can have a significant impact on the objective value, we conduct sensitivity analyses of these parameters in the next section.

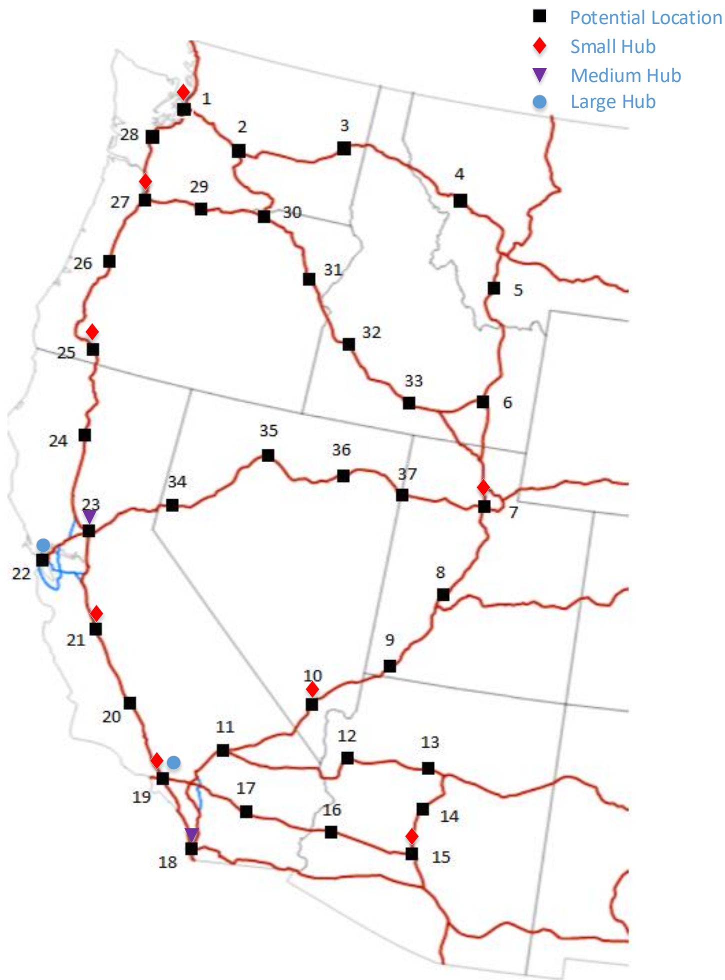


Figure 2.2: Hub location and size decisions in the western interstate highway system

2.4.1.1 Sensitivity analysis of the deterministic model

In this section, sensitivity analysis is performed on the maximum allowable travel time θ, fixed cost, and overtime penalty. Maximum allowable travel time θ can be 5, 7.5, or 10 hours. Under the assumption that a truck driver works 10 hours a day, $\theta = 5$ ensures the driver to come back home within one day, $\theta = 7.5$ restricts the driver to come back home within one and a half days, and $\theta = 10$ limits the driver to come back home within two days (one night on the road). Penalty cost for overtime is set to 50% or 100% of the regular time transportation cost. For example, if the additional penalty is 100% of the regular time transportation cost, then a trucker's salary are doubled during overtime. Fixed cost is another factor that will impact the result. In the case of high penalty cost and fairly low fixed cost, truck drivers will drop off the products after 5 hours travel time since building a hub is not as expensive as working overtime. On the other hand, if penalty cost is low compared to fixed cost, then truck drivers will drive up to θ hours before stopping.

Table 2.2: Selected routing decisions

Flows between Seattle (1) and San Francisco (22)			
$1 \rightarrow 22$	Proportion	$22 \rightarrow 1$	Proportion
$1 \rightarrow 25$	1	$22 \rightarrow 25$	1
$25 \rightarrow 22$	0.844	$25 \rightarrow 1$	0.119
$25 \rightarrow 23$	0.156	$25 \rightarrow 27$	0.881
$23 \rightarrow 22$	0.156	$27 \rightarrow 1$	0.881
Flows between Sacramento (23) and Phoenix (15)			
$23 \rightarrow 15$	Proportion	$15 \rightarrow 23$	Proportion
$23 \rightarrow 21$	1	$15 \rightarrow 19$	1
$21 \rightarrow 19$	1	$19 \rightarrow 23$	1
$19 \rightarrow 15$	1		
Flows between Phoenix (15) and Las Vegas (10)			
$15 \rightarrow 10$	Proportion	$10 \rightarrow 15$	Proportion
$15 \rightarrow 10$	1	$10 \rightarrow 15$	1

Three different hub unit costs are analyzed. In order to test the hub building cost, we introduce another variable ϕ, which measures the ratio of the sum of transportation cost and deadhead cost to the fixed cost. The goal is to find a reasonable hub unit cost so that the overall ratio ϕ can be somewhere between 0.5 to 2. In reality, different companies have different expense structure. A typical pharmaceutical company has ϕ around 5 while retail has ϕ around 0.6. A Chemical company usually has ϕ around 1.5 while high-technology company has ϕ around 2.4 (Alicke and Lösch, 2010). Correlations between θ and penalty cost β, θ and ratio ϕ are shown in Table 2.3 and in Table 2.4, respectively. The second column reports the number of hubs with 10/3/3 indicating that ten small hubs, three medium hubs, and three large hubs are built.

Table 2.3: Correlations between θ and β

	# of hubs	deadhead($)	transportation($)	fixed($)	extra time(hrs)
$\theta = 5$, $\beta = 11$	10/3/3	2.01×10^8	1.12×10^9	1.96×10^9	5.11×10^7
$\theta = 5$, $\beta = 22$	10/3/3	2.01×10^8	1.12×10^9	1.96×10^9	5.11×10^7
$\theta = 7.5$, $\beta = 11$	9/2/2	2.04×10^8	1.11×10^9	1.51×10^9	4.91×10^7
$\theta = 7.5$, $\beta = 22$	9/2/2	2.04×10^8	1.15×10^9	1.51×10^9	4.92×10^7
$\theta = 10$, $\beta = 11$	8/2/2	2.03×10^8	1.13×10^9	1.42×10^9	4.9×10^7
$\theta = 10$, $\beta = 22$	8/2/2	2.04×10^8	1.17×10^9	1.42×10^9	4.93×10^7

In Table 2.3, hub unit cost is set to \$ (90,80,70) per unit of product for small, medium, and large hub, respectively to incorporate economies of scale. Concretely, the fixed cost for building and operating a small hub equals to the capacity of that hub times 90. Only θ and β can vary in Table 2.3. When $\theta = 5$, overtime is actually not allowed since no penalty in the first 5 hours and the maximum allowable travel time is also 5. That is, penalty cost does not affect the decisions

Table 2.4: Correlations between θ and ratio ϕ

	# of hubs	deadhead($)	transportation($)	fixed($)	extra time(hrs)	$\phi \approx$
$\theta = 5$, U_1	9/2/4	2.19×10^8	1.18×10^9	3.84×10^9	5.38×10^7	0.37
$\theta = 5$, U_2	10/3/3	2.01×10^8	1.12×10^9	1.96×10^9	5.11×10^7	0.67
$\theta = 5$, U_3	10/3/3	2.01×10^8	1.12×10^9	9.79×10^8	5.11×10^7	1.35
$\theta = 7.5$, U_1	7/3/2	1.99×10^8	1.19×10^9	2.98×10^9	5.04×10^7	0.47
$\theta = 7.5$, U_2	9/2/2	2.04×10^8	1.15×10^9	1.51×10^9	4.92×10^7	0.9
$\theta = 7.5$, U_3	9/2/2	2.04×10^8	1.15×10^9	7.55×10^8	4.92×10^7	1.79
$\theta = 10$, U_1	8/2/2	2.04×10^8	1.17×10^9	2.84×10^9	4.93×10^7	0.48
$\theta = 10$, U_2	8/2/2	2.04×10^8	1.17×10^9	1.42×10^9	4.93×10^7	0.96
$\theta = 10$, U_3	8/2/2	2.04×10^8	1.17×10^9	7.11×10^8	4.93×10^7	1.93

when $\theta = 5$ and this is the reason why all of the travel expenses are identical in the first two rows in Table 2.3. As we increase θ to 7.5, the number of hubs decreases since truck drivers are allowed to move in a larger route between returns to home base. Setting θ to 10 gives truck drivers even more flexibility. If we compare the hub quantity in Table 2.3, it is obvious that $\theta = 5$ requires more hubs than $\theta = 7.5$ and $\theta = 10$. Although the penalty cost β does not affect hub decisions, the routing decisions are actually different for different combinations of θ and β. Concretely, for the same θ, larger β leads to more extra time since long tour without stops creates penalty cost and short tour with stops creates extra travel time. In order to see the affect of fixed cost, we fixed the transportation cost, penalty cost and deadhead cost parameters and only changed θ and hub unit cost in Table 2.4. Three different combinations of hub unit costs are examined: U_1: \$ (180, 160, 140) , U_2: \$ (90, 80, 70) and U_3: \$ (45, 40, 35) per unit of product. The goal to conduct sensitive analysis on the hub cost is to find a reasonable hub unit cost so that overall transportation cost and fixed cost are in the similar order of magnitude. As hub unit costs decrease (from $U_1 \to U_3$), the system tends to add more hubs and long haul are avoided. Note that when $\theta = 10$, hub and route decisions are unchanged across the different hub unit cost levels. This indicates that the system is really robust for reasonable hub unit cost. However, hub and route decisions are still sensitive when the hub unit costs are extremely small. By changing parameters θ and hub unit cost, we obtain ϕ ranging from 0.37 to 1.93 meaning transportation cost and fixed cost are comparable in terms of magnitude. One significant improvement in Table 2.3 and Table 2.4 is the deadhead cost. The traditional PtP transportation system ends up with deadhead cost equal to \$ 4.4×10^8, which is twice as much as the deadhead cost in our model due to the fact that truck drivers are much more flexible in the PI-inspired hub network.

To test the computational viability of the model, the full 37 node network was solved for a variety of flow levels. All computations were performed on Intel(R) Core(TM)2 Quad CPU and results are shown in Table 2.5. The number of binary variables remains unchanged since the quantity of flows does not affect the hub location and size options. The number of flows and complexity of the problem increase as we add shipment sources (suppliers) and destinations (customers). Initially selecting 10 origins and destinations produces 90 flows. The number of customers and suppliers then increments by 2 in each row of Table 2.5. Note that the computation time increases exponentially from approximately 1 minute to 81 hours. But as this is a design problem, the model is still tractable for a moderate network size.

Table 2.5: Computation times and model statistics for the deterministic model

# of flows	# of binary	# of continuous	# of equations	computation time
90	111	124,695	67,633	1 min 21 sec
132	111	182,189	97,911	18 mins 23 sec
182	111	250,639	133,961	22 mins 13 sec
240	111	330,041	175,779	1 hrs 05 mins
306	111	420,395	223,365	2 hrs 18 mins
380	111	521,701	276,719	15 hrs 14 mins
462	111	633,959	335,841	81 hrs 9 mins

2.4.2 Results of two-stage stochastic programming model

The uncertainty under consideration is daily demand between suppliers and customers. A two-stage stochastic programming model is formulated to design the system under uncertainty. Hub sizes and locations are first determined. The routing problem is then solved each period based on the daily volumes. To test the model we generate a set of daily demand scenarios that represent possible demand realizations. The mean of those scenarios is set equal to the daily demand in the deterministic model. The standard deviation is set to 10% or 20% of mean demand in order to gauge the reaction of the system. Demand is assumed to be normally distributed and 10 possible realizations of demand are generated. The comparison of the deterministic model and stochastic model is included in Table 2.6. The first row in Table 2.6 conveys the results from the deterministic model. The rest of the results are from the stochastic programming model. As the standard deviation increases, fixed cost and the objective value increase as the system tends to build hubs with larger capacity. When the standard deviation increases to 10% and then 20% of the mean, one small hub and then two small hubs are replaced by medium hubs, respectively. Intuitively, larger hubs require more expenses but provide system with more flexibility against uncertainty. The details of hub location decisions are included in Table 2.7. One of our modeling assumptions is that each customer and supplier must have sufficient storage to receive and ship out products. Since we only change the volume of demand while keeping locations of customers and suppliers unchanged, our hub location decisions are not particularly sensitive to the uncertainty parameter. Hub capacities are impacted however. Notice that a small hub is upgraded to a medium hub at site 19 when the standard deviation of demand increases to 10% and another change is made at site 15 when the standard deviation of demand increases to 20%.

In order to test whether the stochastic model provides better results than the deterministic model, we conduct analyses from two different perspectives: (i) Identifying the bottleneck of the deterministic model; (ii) Measuring the Expected Value of Perfect Information (EVPI) . We start the analysis by testing the robustness of the hub decisions in the deterministic model. We fix the hub decisions at the values obtained from the deterministic model and then solve the deterministic model for each scenario. The details are shown in Table 2.8. The main motivation to test the robustness of the hub decision is that in the stochastic programming model only the first stage

Table 2.6: Comparison of the deterministic model and stochastic model

	# of hubs	objective value($)	(expected) trans' cost($)	fixed cost($)	(expected) extra time(hrs)
std=0%	8/2/2	7,670,668	3,212,719	3,898,200	124,139
std=10%	7/3/2	7,879,616	3,231,276	4,085,100	124,702.4
std=20%	6/4/2	8,084,275	3,238,207	4,272,000	124,733.6

Table 2.7: Hub location decisions in the stochastic model

	std=0%			std=10%			std=20%		
Location	Small	Medium	Large	Small	Medium	Large	Small	Medium	Large
1	1			1			1		
7	1			1			1		
10	1			1			1		
15	1			1				1	
18		1			1			1	
19	1		1		1	1		1	1
21	1			1			1		
22			1			1			1
23		1			1			1	
25	1			1			1		
27	1			1			1		

decision is implementable since it is scenario independent. Anything beyond the root node is scenario dependent. Intuitively, if all of the scenarios are feasible, the deterministic model is capable of handling the uncertainty. Otherwise, the stochastic model overrides the potentially infeasible in practice deterministic model solution. Using the hub decisions from the deterministic model, when the standard deviation is 10%, we obtain only one infeasible scenario which is scenario 3. When the standard deviation is 20%, this number increases to three which is a sign that the deterministic model becomes more and more vulnerable as variation increases. Table 2.8 and 2.9 aim to provide insights on the capacity expansion investment. For example, when the standard deviation is 10% we obtain an optimal solution by upgrading a small hub at site 19 to a medium hub. On the other hand, we can find feasible solutions for every scenario by expanding the capacity of small hubs or large hubs by 64 units. The first type of investment is an optimal strategy in terms of the model's perspective but the second is cheaper in terms of fixed cost investment and indicates a potentially more meaningful use of the model. In the third column of Table 2.8, we measure the slack capacity, that is, how much additional capacity we need to invest in order to make the problem feasible. Since we only change the volume of demand, and the locations of suppliers and customers stay the same, we can ensure a feasible solution by increasing the capacity of hubs. Note that we have three options when it comes to capacity investment: small hub, medium hub, and large hub. In Table 2.8, 64 / ∞ / 64 means if we only have a chance to increase the capacity of small hubs, then we need 64 additional units of capacity in order to ensure a feasible solution. Likewise, if we only have a chance to increase the capacity of large hubs, then we need 64 units as well. Note that the slack of medium hub is ∞ due to the fact that medium hubs are not the bottleneck in scenario 3 and increasing the capacity of medium hubs is not going to achieve feasibility. Scenarios 7, 8, and 10 are infeasible when the standard deviation is 20%. Respectively, 319 and 266 units of capacity are needed for small hubs if we want to obtain a feasible solution to scenario 7 and scenario 8. For scenario 10, in order to maintain feasibility we can either increase the capacity of small hub or the capacity of large hub by 354. Clearly, the deterministic hub decision becomes more and more unreliable as we introduce more variations and it will be reflected in the results of EVPI.

Table 2.8: Robustness test of the hub decisions in the deterministic model

Standard deviation = 10%			
Scenario#	Objective value($)	Feasibility	Slack
1	7,689,447	✓	-
2	7,673,031	✓	-
3	-	x	64 / ∞ / 64
4	7,670,460	✓	-
5	7,744,635	✓	-
6	7,700,213	✓	-
7	7,643,186	✓	-
8	7,742,715	✓	-
9	7,724,907	✓	-
10	7,674,543	✓	-
Standard deviation = 20%			
Scenario#	Objective value($)	Feasibility	Slack
1	7,698,681	✓	-
2	7,575,240	✓	-
3	7,395,544	✓	-
4	7,735,208	✓	-
5	7,682,301	✓	-
6	7,716,281	✓	-
7	-	x	319 / ∞ / ∞
8	-	x	266 / ∞ / ∞
9	7,766,382	✓	-
10	-	x	354 / ∞ / 354

In addition, we investigate the bottleneck of this system, that is, which hubs should be consolidated if we want a stronger and more robust network system. The bottleneck of the system can be tested by recording the volume of flow through each hub. Cities without any hubs can be ignored at this moment because of flow conservation. Since there is 1 infeasible case when the standard deviation is 10% and 3 infeasible cases when the standard deviation is 20%, we decide to pay more attention to identify the bottleneck of the system for those infeasible cases. The details of the bottleneck are shown in Table 2.9.

Table 2.9: Identifying the bottleneck of the network system

Standard deviation	Scenario#	Bottleneck
10%	3	10,19,21,27
20%	7	19,21,27
20%	8	15,19,21,27
20%	10	10,19,21,27

Different cases have different bottlenecks due to the volume of demand. Hubs at sites 19, 21 and 27 are the bottlenecks of all 4 cases meaning they have higher priority for investment if we want to upgrade and maintain a robust system. One interesting observation is that the slack of small hub is exactly the same as the slack of large hub in scenario 3 and scenario 10 meaning we can upgrade either small hubs or large hubs in order to get a feasible solution. Essentially we just need additional capacity along the path containing those hub locations and the continuous routing variables provide the flexibility to use the capacity allocation.

Next we analyze the model from a stochastic point of view by calculating several common measures including here-and-now (RP), wait-and-see (WS), and expected value of perfect information (EVPI). RP value is just the objective value of the stochastic model. WS is the weighted objective value of reacting with perfect foresight, that is, we solve each scenario in the deterministic model and take the expected value based on their scenario probabilities and respective objective values. EVPI measures the difference between RP and WS, that is, how much can we improve the quality of decision if we have perfect information, or, in our case, we had known static flow volumes. EVPI = 158,485 and 298,033 for standard deviation 10% and 20%, respectively. As expected, the more variations in the model, the more valuable the information. The computational viability of the stochastic model is reported in Table 2.10. The allowable optimality gap is set to 1% except for the first model with 90 flows that is solved to optimality.

Table 2.10: Computation times and model statistics for the stochastic model

# of flows	# of binary	# of continuous	# of equations	computation time
90	111	1,245,902	676,281	54 mins
132	111	1,820,882	979,341	2 hrs 16 mins
182	111	2,505,382	1,339,881	2 hrs 27 mins
240	111	3,299,402	1,758,101	4 hrs 10 mins
306	111	4,202,942	2,233,641	5 hrs 11 mins
380	111	5,216,002	2,767,181	6 hrs 28 mins

* Standard deviation is 10%

2.4.3 Computation comparison

In order to obtain a high quality solution faster, two different sets of feasibility cuts are added. In order to explain these cuts, we first define sets $S_i^o = \{j : t_{ij} \leq \theta, j \neq i\}$ and $S_l^o = \{j : t_{jl} \leq \theta, j \neq l\}$ representing the feasible one step jumps from sources and into destinations, respectively. Two feasibility cuts added in the model are as follows:

$$\sum_z \sum_{j \in S_i^o} U_{jz} \cdot Y_{jz} \geq \sum_{l, l \neq i} w_{il} \qquad \forall \{i\} \tag{2.26}$$

$$\sum_z \sum_{j \in S_l^o} U_{jz} \cdot Y_{jz} \geq \sum_{i, i \neq l} w_{il} \qquad \forall \{l\} \tag{2.27}$$

Equation 2.26 and 2.27 ensure construction of hubs near source and sink locations, ie. within a feasible trip distance, with sufficient capacity for accommodating all flow out from sources and into sinks. We compared computation times in Figure 2.3 using the feasibility cuts mentioned above. Notice that about 30% of computation time can be saved using these feasibility cuts when the number of selected cities is less than 20. Future research could explore developing and implementing additional cuts for Benders Decomposition and the L-shaped method. An appropriate problem for Benders Decomposition is a problem with many continuous variables and relatively few integer variables. The separable continuous-variable, routing subproblems for each scenario provide a natural platform for applying the method.

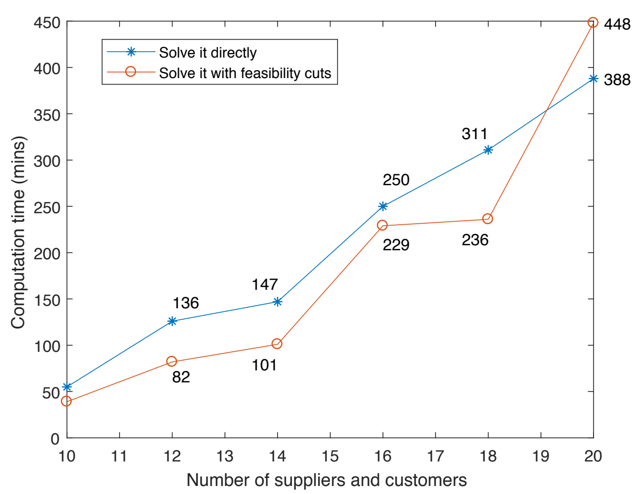


Figure 2.3: Analysis on the computation time

2.5 Conclusions

Long haul trucking constitutes a major segment of the U.S. logistics system. Long intercity distances and the desire to reduce deadheading yield long routes that keep drivers on the road for extended periods. That results in high driver turnover and intermittent movement of freight. A hub-based relay network is proposed wherein drivers can normally return to their base on a daily basis while coordinated scheduling would allow more continuous movement of freight and potentially improved city logistics for final delivery. Deterministic and stochastic demand versions of a mixed integer programming model are developed to determine the location of hubs, hub capacity, and load routing. The models are shown to be computationally feasible for mid-sized problems. Network topology is shown to be sensitive to uncertainty but more so in the sizing of hubs than the location. It appears advisable to add sufficient capacity to the hubs to cover potential variability in the shipping volume.

On top of that, network topology is also sensitive to various parameters such as maximum trip duration, overtime penalty for multi-day trips and hub unit cost. Robustness of, and bottlenecks in, the stochastic system are examined. The system is found to become more and more vulnerable as variation increases. Bottleneck and slackness tests provide an alternative approach to upgrade the system. Feasibility cuts based on hub capacity are generated to test computation efficiency. The model with additional feasibility cuts works well and saves around 30% of computation time when the number of flows is less than 380.

Our study is subject to a few limitations which suggest future research directions: Firstly, developing additional good feasibility cuts that can be added in the master problem of L-shaped method. Secondly, Instead of using 10 scenarios to approximate the uncertain demand, we can have more scenarios using Sample Average Approximation (SAA). Thirdly, in-sample and out-of-sample stability tests can be conducted to validate the effectiveness of the approximation. Concretely, we can generate different set of scenarios and solve the stochastic problem with first-stage decision variables either fixed or not fixed. The effectiveness of the approximation can be evaluated by comparing the optimal objective values. Last but not least, heuristics can be used when solving large scale problem instances. There can be two stages in the heuristic algorithms: hub decision stage and routing decision stage. In hub decision stage, we can add one hub a time and monitor the changes in the objective value. Alternatively, we could begin with a dense set of hubs and iteratively remove hubs in a greedy fashion. In the routing decision stage for a given supplier and customer, there can be multiple routes constructed from different highway segments or using different hub sequences. If the system chooses a particular segment of highway or hub, then all routes that do not include that segment of highway or hub should be eliminated.

CHAPTER 3. A MULTI-STAGE STOCHASTIC PROGRAMMING FOR LOT-SIZING AND SCHEDULING UNDER DEMAND UNCERTAINTY

A stochastic lot-sizing and scheduling problem with demand uncertainty is studied in this paper. Lot-sizing determines the batch size for each product and scheduling decides the sequence of production. A multi-stage stochastic programming model is developed to minimize overall system costs including production cost, setup cost, inventory cost and backlog cost. We aim to find the optimal production sequence and resource allocation decisions. Demand uncertainty is represented by scenario trees using moment matching technique. Scenario reduction is used to select scenarios with the best representation of original set. A case study based on a manufacturing company has been conducted to illustrate and verify the model. We compared the two-stage stochastic programming model to the multi-stage stochastic programming model. The major motivation to adopt multi-stage stochastic programming models is that it extends the two-stage stochastic programming models by allowing revised decision at each period based on the previous realizations of uncertainty as well as decisions. Stability test and weak out-of-sample test are applied to find an appropriate scenario sample size. By using the multi-stage stochastic programming model, we improved the quality of solution by 10% - 13%.

3.1 Introduction

Production planning aims to determine the best allocation of production resources to meet demand over a time period with a limited amount of production capacity. Based on the length, production planning decisions can be categorized into three different terms: long-term, medium-term and short-term. Facility location design and resource allocation are considered as long-term decision making problems. Medium-term planning considers production quantity on a monthly basis, and short-term planning involves making decisions such as day-to-day schedule of activities and job sequencing. In the classical hierarchical decision-making environment, lot-sizing and scheduling decisions take place in the medium-term planning levels that usually span about half an year (Karimi et al., 2003; Özdamar and Birbil, 1998). Over the entire production horizon, the market and manufacturing configurations might change and therefore, considering uncertainty and designing a robust production plan are crucial in the lot-sizing and scheduling problem.

The major motivations for this paper can be summarized as follows: First, lot-sizing and scheduling problems have been widely applied in industry. Gupta and Magnusson (2005) studied a lot-sizing and scheduling problem confronted by a large manufacturing company that produces sandpaper rolls of different grades or roughness. Bitran and Gilbert (1989) reviewed the lot-sizing and scheduling problem using a chemical application. In the field of chemicals, setup cost takes place when it is necessary to scrub out a machine between the production of two products that come from different families. Silva and Magalhaes (2006) focused on a lot-sizing and scheduling problem in a company that produces acrylic fiber for textile industry. The problem arises because a changeover occurs between two lots of products due to tool wear. Second, little attention has been paid to stochastic lot-sizing and scheduling problem, especially, multi-stage stochastic lot-sizing and scheduling problem. The Lot-sizing and scheduling problem is an extension of lot-sizing problem which considers production sequence. Harris (1990) introduced a single-item lot-sizing model with deterministic static demand. The goal is to minimize overall costs include ordering and inventory. Brahimi et al. (2006) reviewed both uncapacitated and capacitated single item lot-sizing problem. Different mathematical formulations and extensions of real world applications are studied. The problem we focus in this paper is called the capacitated lot-sizing and scheduling problem with sequence dependent setups (CLSD). It is a variation of the CLSP which incorporates dependent setups. Kaczmarczyk (2011) considered a lot-sizing and scheduling problem that allows only one setup in each time period. Their formulation includes multi-product and identical parallel machines. Kimms (2012) proposed a multi-level lot-sizing and scheduling problem with dynamic demand. Multi-level production means the final product in one stage can be used as raw material in the next stage. However, these papers considered only the deterministic lot-sizing and scheduling problems which may not reflect the reality. This point serves as a major motivation for this research.

Production plan can be highly affected by the various uncertainties such as yield, demand and defective rate. Alem et al. (2018) used the lot-sizing and scheduling problem as an application to compare the performances and results of stochastic approach with robust optimization approach. The advantage of each approach was assessed via a Monte Carlo simulation procedure. Rahdar et al. (2017) proposed a two-stage trilevel optimization model with a rolling horizon. Demand and lead time uncertainty are studied. Hu and Hu (2016) proposed a two-stage stochastic programming approach for the lot-sizing and scheduling problem under demand uncertainty. They proved that the stochastic model outperforms the deterministic model and considering uncertainty is important. Ramaraj et al. (2017) studied multiple uncertain parameters using a two-stage stochastic programming model. However, the main drawback of the two-stage stochastic programming technique is it does not take into account the sequential decision making due to the multiple periods in the planning horizon. That is, all of the resource decisions have to be done by the beginning of the second period and no makeup/corrective decision is allowed when new information is revealed (Hu et al., 2017). Unlike the two-stage model, the multi-stage stochastic programming model explicitly addresses and incorporates the sequential relationship of the decisions over the multiple periods in the planning horizon (Li et al., 2006b). The trade-off is that the computational complexity of the multi-stage stochastic programming model is much higher compared to the two-stage stochastic programming model. Therefore, the problem size that can be solved is limited. Huang and Ahmed (2009) compared the two-stage model with multi-stage model and used heuristics to derive the bound for the value of multi-stage stochastic programming (VMS). The results show that even a feasible solution for the multi-stage model can be much better than the optimal solution for the two-stage model.

The major contributions of this study can be summarized as follows: Firstly, we proposed a novel multi-stage stochastic programming model to deal with demand uncertainties. We demonstrated that for a multi-period problem, it is more suitable to use a multi-stage stochastic model. Secondly, stability test is used to identify the best scenario size, which is a significant improvement from the existing literature. Thirdly, we quantitatively measured the improvement of results using multi-stage stochastic programming model. Finally, we provided guidelines to choose the most suitable approach for decision makers based on the results and computational performance.

The remainder of this paper is organized as follows: problem statement and model formulations for both deterministic model and multi-stage stochastic programming model are presented in section 3.2. The numerical results and model comparisons are reported in section 3.3. Finally, conclusions, limitations, and future works are discussed in section 3.4.

3.2 Model formulation

The deterministic model and multi-stage stochastic programming model are introduced in this section. In a lot-sizing and scheduling problem, each time slot typically represents a week or a month while the overall production horizon is usually no longer than half an year (Guimarães et al., 2014; Drexl and Kimms, 1997). We aim to find the best production decisions such that the overall cost is minimized. The deterministic model is presented followed by the multi-stage stochastic programming model in which demand uncertainty is considered. There are two types of decisions in the multi-stage model: regular time production decisions and recourse decisions. The regular time production decisions need to be determined at the beginning of each time slot while the recourse decisions include overtime production, inventory, and backlogs are made after the realization of uncertainty in the current stage.

3.2.1 Problem statement

The problem we address in this paper can be described as follows: manufacturers acquire raw material from up-stream suppliers and produce final products for downstream plants or customers. Orders can be placed at the beginning of each month. According to resource availability, decision makers need to design a good production plan so that the costs can be minimized. Two different resource capacities are: time capacity on the machine and production quantity limitation. Unmet demand can be fulfill later since backlog is allowed. Decision variables include regular time production, overtime production, production sequence, inventory and backorder. The regular time production is limited by both the time capacity on the machine and resource availability. Overtime production is proportional to the regular time production. Production sequence is really critical because setup is sequence dependent and can be carried over to the following period. In other words, different production sequences will result in different resource requirements. Inventory and backorder can then be evaluated.

In the deterministic model, parameters are fixed and known. In the stochastic model, demand is uncertain and represented by scenarios. In the stochastic model, regular time production and production sequence need to be determined in the presence of uncertainty while overtime production decision are made after uncertainty is realized. Scenario sample size analysis and weak out-of-sample stability test aim to identify a good scenario sample size. The analysis of the two-stage stochastic programming model demonstrates the importance of considering uncertainty. The comparisons between two-stage and multi-stage stochastic programming models include computation time and objective value.

3.2.2 Mathematical notations

The mathematical notations for the deterministic model are included in Table 3.1. These parameters are fixed and known in the deterministic model. The parameters and variables in the stochastic model are scenario-based and the means of those parameters are same as the deterministic model.

3.2.3 Model assumptions

The assumptions are listed as follows:

- Inventory and backlog are allowed which indicate demand does not need to be fulfilled all the time. The initial values of inventory and backlog are assumed to be zero.

- Demand is time independent, so the realization of demand in current stage does not depend on the previous realization.

- The uncertain demand is realized at the beginning of each period, inventory and backorder levels will be measured at the end of each production period.

- The regular time production and overtime production are resources limited. The former has time and batch size capacities while the latter only has batch size limitation.

- The regular time production and overtime production share the same setup. Since the actual demand is realized after production started, the overtime production serves as the recourse for the baseline production.

- A setup is required between products from different families. In addition, a setup can be carried over between two consecutive production periods. Therefore, the last setup in one period will be the first default setup in the following period.

3.2.4 Deterministic model

The deterministic model aims to minimize the overall system costs, including regular time production cost, overtime production cost, setup cost, inventory cost, and backlog cost. The first and second terms in the objective function are the regular time production cost and overall setup changeover cost, respectively. It should be noted that there is no setup between products from the same family. The third term is the overtime production cost. The last two terms are the overall inventory holding cost and the backlog cost, respectively. There are three possible cases for the inventory and the backlog costs. First, they all equal to zero meaning current demand is met and no extra product is manufactured. Second, inventory is positive and backlog is zero indicating current demand is met and extra products are manufactured for future demand. Third, inventory is zero and backlog is positive showing that the production capacity is not sufficient to satisfy the demand requirement. The last production time is not included in this model because we add it as a dummy period.

$$\begin{aligned}\min \zeta = &\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^r * X_{i,t} + \sum_{i=1}^{I}\sum_{i\neq j}\sum_{j=1}^{J}\sum_{t=1}^{T} sc_{i,j} * Y_{i,j,t} \\ &+ \sum_{i=1}^{I}\sum_{t=1}^{T} p_i^o * O_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T} h_i * I_{i,t} + \sum_{i=1}^{I}\sum_{t=1}^{T} b_i * B_{i,t}\end{aligned} \tag{3.1}$$

3.2.4.1 Constraints of deterministic model

Equation 3.2 and 3.3 are product flow conservation constraints. The total amount of production plus inventory from the previous time period equal to the total demand plus the current inventory. The inventory in these two constraints can be either the extra inventory or the backlog demand

Table 3.1: Notations for the deterministic model

Subscripts		
i	$1, 2 \cdots N$	Material index
j	$1, 2 \cdots N$	Material index
t	$1, 2 \cdots T + 1$	Time period index
Parameters		
$d_{i,t}$	Demand of material i at time t	
h_i	Holding cost of each material i for one time period	
b_i	backorder cost of each material i for one time period	
cap_t	Time capacity on the machine at time t	
p_i	Manufacturing time of each material i	
p_i^r	Regular time manufacturing cost of each material i	
p_i^o	Overtime manufacturing cost of each material i	
$q_{i,t}$	The maximum regular time batch size of product i at t	
$sc_{i,j}$	Setup changeover cost from material i to j	
$st_{i,j}$	Setup time from material i to material j	
N	Number of material families	
Decision Variables		
$I_{i,t}$	Inventory quantity of material i at the end of time t	
$B_{i,t}$	Backorder quantity of material i at the end of time t	
$X_{i,t}$	Regular production quantity of material i during time t	
$O_{i,t}$	Overtime production quantity of material i during time t	
$Y_{i,j,t}$	Binary variable takes value 1 if there is a setup changeover from material i to material j during time t	
$Z_{i,t}$	Binary variable takes value 1 if setup of material i carried over from previous time slot to time t	
$V_{i,t}$	Sequence of production in time period t	

that can be fulfilled later. There is no inventory coming into the first time period as we assume that initial inventory and backlog are both zero.

$$X_{i,t} + O_{i,t} = d_{i,t} + I_{i,t} - B_{i,t} \qquad \forall i, t = 1 \tag{3.2}$$

$$I_{i,t-1} - B_{i,t-1} + X_{i,t} + O_{i,t} = d_{i,t} + I_{i,t} - B_{i,t} \qquad \forall i, t = 2 \cdots T + 1 \tag{3.3}$$

Equation 3.4 restricts that the regular time production quantity will not exceed the maximum regular time production quantity which is $q_{i,t}$. Recall that only one setup is allowed in each product family indicating $Z_{i,t} + \sum_{j \neq i}^{J} Y_{j,i,t} \leq 1$. If both terms are zero, then product i cannot be manufactured in time period t. If $Z_{i,t} = 1$ and $\sum_{j \neq i}^{J} Y_{j,i,t} = 0$, then material i will be the first product on the assembly line. If $Z_{i,t} = 0$ and $\sum_{j \neq i}^{J} Y_{j,i,t} = 1$, then material i will be manufactured after material j.

$$X_{i,t} \leq q_{i,t} * (Z_{i,t} + \sum_{j \neq i}^{J} Y_{j,i,t}) \qquad \forall i, t \tag{3.4}$$

Total machine time capacity, denoted by cap_t, is the maximum regular time resource on the machine. Equation 3.5 ensures that the total time for the regular production and setup changeover time cannot go beyond the total machine time capacity. Equation 3.6 sets a capacity limit on the overtime production quantity. Typically, $\alpha * X_{i,t}$ puts a production quantity capacity on the overtime production (Zhang et al., 2011).

$$\sum_{i=1}^{I} p_i * X_{i,t} + \sum_{i=1}^{I} \sum_{i \neq j \, j=1}^{J} st_{i,j} * Y_{i,j,t} \leq cap_t \qquad \forall t \tag{3.5}$$

$$O_{i,t} \leq \alpha * X_{i,t} \qquad \forall i, t \tag{3.6}$$

Equation 3.7 states at the beginning of each time period, a setup is carried over from the previous time period. Equation 3.8 states that the setup flow going into material i equals to the one coming out of it. One easy example will be producing the same product during the entire production horizon meaning $Z_{i,t} = Z_{i,t+1} = 1$ and all of the Y variables are zero because there is no setup changeover.

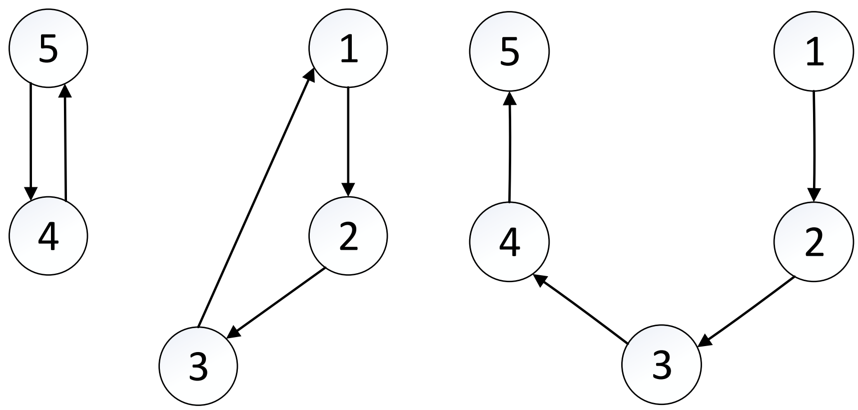
$$\sum_{i=1}^{I} Z_{i,t} = 1 \qquad \forall t \tag{3.7}$$

$$Z_{i,t} + \sum_{j \neq i}^{J} Y_{j,i,t} = Z_{i,t+1} + \sum_{j \neq i}^{J} Y_{i,j,t} \qquad \forall i, t = 1 \cdots T \tag{3.8}$$

Equation 3.9 requires that no production activity is allowed in the last dummy period except that the setup is carried over from the previous time period. Equation 3.10 is one of the sub-tour elimination constraints which has been widely applied in the traveling salesman problem. It enforces that there is only a single tour covering all the given nodes and no disjointed tours are allowed. Figure 3.1a shows an example of sub-tour. A disjointed tour (1-2-3-1) is not allowed in the Traveling Salesman Problem (TSP). Equation 3.10 makes sure that each product will be visited once and only once. The feasible route in Figure 3.1b assigns $V_{1,t} = 1$, $V_{2,t} = 2, \cdots, V_{5,t} = 5$.

$$X_{i,t} = 0 \qquad \forall i, t = T + 1 \tag{3.9}$$

$$V_{j,t} \geq V_{i,t} + 1 - N * (1 - Y_{i,j,t}) \qquad \forall i, j \neq i, t \tag{3.10}$$



(a) An example of sub-tour (b) An example of feasible route

Figure 3.1: A sub-tour and feasible solution to the traveling salesman problem

3.2.5 Multi-stage stochastic programming model

In this study, variables $X_{i,t}$, $Y_{i,j,t}$, $Z_{i,t}$, and $V_{i,t}$ are the baseline production decisions which involve the regular time production quantity and sequence of production. Decision variables $O_{i,t,s}$, $I_{i,t,s}$, and B_{its} are recourse decisions. Demand is the uncertain factor under investigation since it is among the most common uncertain factors in the production design problem. Uncertainties are usually represented with discrete probabilistic scenarios since continuous distributions are computationally challenging to implement in the model. We use a number of scenarios, i.e, $S = \{\mu_1, \cdots, \mu_s\}$ and corresponding probability ν_s to represent original distribution. Each realization represents the demand in that particular time period while the series of realizations disclose the evolution of uncertain demand. In each time period, multiple realizations will be generated to capture the statistical properties of continuous distribution. The multi-stage stochastic programming model aims to design a production planning with uncertain demand considered explicitly. All of the variables in the deterministic model need to be slightly changed by adding a scenario index s which have probability ν_s. The multi-stage stochastic programming model is formulated as follows:

$$\begin{aligned}\min \zeta = \sum_{s=1}^{S} \nu_s * (\sum_{i=1}^{I}\sum_{t=1}^{T} p_i^r * X_{i,t,s} + \sum_{i=1}^{I}\sum_{i\neq j}\sum_{j=1}^{J}\sum_{t=1}^{T} sc_{i,j} * Y_{i,j,t,s} \\ + \sum_{i=1}^{I}\sum_{t=1}^{T} p_i^o * O_{i,t,s} + \sum_{i=1}^{I}\sum_{t=1}^{T} h_i * I_{i,t,s} + \sum_{i=1}^{I}\sum_{t=1}^{T} b_i * B_{i,t,s})\end{aligned} \tag{3.11}$$

$$X_{i,t,s} + O_{i,t,s} = d_{i,t,s} + I_{i,t,s} - B_{i,t,s} \qquad \forall i, t = 1, s \tag{3.12}$$

$$I_{i,t-1,s} - B_{i,t-1,s} + X_{i,t,s} + O_{i,t,s} = d_{i,t,s} + I_{i,t,s} - B_{i,t,s} \qquad \forall i, t = 2 \cdots T + 1, s \tag{3.13}$$

$$X_{i,t,s} \leq q_{i,t} * (Z_{i,t,s} + \sum_{j \neq i}^{J} Y_{j,i,t,s}) \qquad \forall i, t, s \tag{3.14}$$

$$\sum_{i=1}^{I} p_i * X_{i,t,s} + \sum_{i=1}^{I} \sum_{i \neq j\ j=1}^{J} st_{i,j} * Y_{i,j,t,s} \leq cap_t \qquad \forall t, s \tag{3.15}$$

$$O_{i,t,s} \leq \alpha * X_{i,t,s} \qquad \forall i, t, s \tag{3.16}$$

$$\sum_{i=1}^{I} Z_{i,t,s} = 1 \qquad \forall t, s \tag{3.17}$$

$$Z_{i,t,s} + \sum_{j \neq i}^{J} Y_{j,i,t,s} = Z_{i,t+1,s} + \sum_{j \neq i}^{J} Y_{i,j,t,s} \qquad \forall i, t = 1 \cdots T, s \tag{3.18}$$

$$X_{i,t,s} = 0 \qquad \forall i, t = T + 1, s \tag{3.19}$$

$$V_{j,t,s} \geq V_{i,t,s} + 1 - N * (1 - Y_{i,j,t,s}) \qquad \forall i, j \neq i, t, s \tag{3.20}$$

Equation 3.11 along with Equation 3.12 - 3.20 are based on the deterministic model by adding the scenario index s to the equations in the deterministic model. The only difference is that baseline production can be determined at the beginning of each time period given previous realizations of uncertainty. It should be noted that there are T time periods in the model and production process. The reason to include last dummy time period is to capture the setup carried over from T to $T+1$. We assume that there is no demand or production in the last time period. If $T + 1$ is not added, then Equation 3.8 and 3.18 will be violated. Besides Equation 3.11 - 3.20, we need an additional type of constraint in the multi-stage model, called non-anticipativity constraints.

As mentioned before, continuous distribution is computationally challenging to implement. Therefore, the uncertainty was approximated with multiple scenarios. The goal of this process is to simplify the problem as well as capture the statistical properties of original distribution (Høyland and Wallace, 2001). In this paper, the planning horizon has multiple periods which significantly increases the number of scenarios. A scenario reduction procedure has been implemented to identify a representative subset of scenario so that essential features and computational tractability can be maintained (Heitsch and Römisch, 2003). The details of moment matching and scenario reduction techniques are discussed in the detail in the case study section.

3.3 Case study

In order to demonstrate and validate the multi-stage stochastic programming model proposed in this paper, we apply a case study to a braking equipment manufacturing plant located in Italy. An analysis shows that disturbances affect both upstream and downstream manufactories. Hence, a robust production design is required to balance between production profit and customer satisfaction. In this case study, the manufacturing plant collects two different types of raw material, P_1B and P_2B, and produces three final braking productsP_1, P_2 and P_3. The overall production horizon for the lot-sizing and scheduling problem is usually shorter than half an year, while each time slot is commonly on the weekly or monthly basis (Guimarães et al., 2014; Drexl and Kimms, 1997).

3.3.1 Data sources

The case study in this paper is a single-level, multi-product, and multi-period stochastic programming problem. Single-level means there is no semi-finished product. Probability density functions of demand are fitted with historical data. Demands of three final products P_1, P_2, and P_3 follow Weibull distribution. We assume that the demand is both product and period independent. The details of statistical properties are shown in Table 3.2.

Table 3.2: Statistical properties of monthly demand

Properties	P_1	P_2	P_3
PDF	Weibull	Weibull	Weibull
Scale	518	38	169
Shape	1.51	2.76	2.27
Mean	467.25	33.82	149.70
Variance	99422	175.4231	4877.8
Skewness	1.06	0.25	0.47
Kurtosis	4.35	2.78	2.98

Production sequences can make significant impact on overall production cost as changeovers are sequence dependent. Therefore, it becomes essential to identify the optimal production plan. Setup changeover time and manufacturing time are listed in Table 3.3 and 3.4, respectively. Setup cost can be derived by multiplying setup time with a constant factor (James and Almada-Lobo, 2011).

Table 3.3: Setup changeover time (mins/setup)

	P_1	P_2	P_3
P1	0	270	90
P2	180	0	270
P3	90	180	0

Table 3.4: Costs and manufacturing time for different products

	P_1	P_2	P_3
Manufacturing time (mins/unit)	6	6.6	7.2
Inventory cost ($/unit month)	0.16	0.15	0.38
Regular production cost ($/unit)	254.08	254.08	254.08

Inventory costs and time capacities are included in Table 3.4 and 3.5, respectively. Gnoni et al. (2003) claimed that time capacity is the bottleneck and critical resource in production. The regular time production costs are shown in Table 3.4. The overtime manufacturing cost and backlog cost are based on the regular manufacturing cost (Zhang et al., 2011; Rego and Mesquita, 2015). Maximum overtime production quantity is setup to 20% of regular production quantity as large overtime allowance reduces efficiency and increases the chance of injury.

Table 3.5: Time capacities on the machine (mins)

Month	Capacity
1	6087
2	5367
3	6087
4	6087
5	4407
6	4407

Identifying the optimal production quantity as well as sequence are two critical decisions in production problems. Park (2005) did a sensitivity analysis to explore the impact of production capacity resource on production decisions. In this study, similar experiment settings have been employed.

3.3.2 Scenario generation and reduction

Representing uncertain parameters with continuous distributions has proven to be computationally challenging for a stochastic model (Feng and Ryan, 2013). A common way to simplify and approximate the continuous distribution is to discretize it with a number of realizations. This process is called scenario generation. Scenario size increases dramatically as the number of time horizons increase which affects the tractability of the solution. Therefore, it is common to select a subset of representative scenarios from the entire set. This process is known as scenario reduction.

3.3.2.1 Scenario generation technique

Scenario generation technique is briefly reviewed in this section. Ψ includes all the statistical properties we want to consider in the model. In this study, ψ belongs to the set Ψ which includes the first four moments. ω_ψ is the weight for statistical property ψ which measures the importance of matching mathematical expression (Heitsch and Römisch, 2003). $f_\psi(\pi, Pr)$ represents the mathematical expression for each ψ, and VAL_ψ is the input parameter for ψ. The goal of this model is to generate the discrete realizations π_ψ with probabilities Pr_ψ so that the squared differences between mathematical expression and given input is minimized. For example, if we want to approximate a normal distribution, then Ψ contains statistical properties such as mean and variance. VAL_ψ is the given mean/variance of the normal distribution as a input parameter. $f_\psi(\pi, Pr)$ is the mathematical expression for mean/variance which can be expressed as $\sum_\psi \pi_\psi Pr_\psi$ or $\sum_\psi Pr_\psi * (\pi_\psi - \sum_\psi \pi_\psi Pr_\psi)^2$. Equation 3.21 aims to minimize the overall weighted squared distance between the specified value of the statistical property and the value of the mathematical expression. An objective value of zero means that the discrete realizations match with the specified statistical property perfectly. Equation 3.22 and 3.23 state that the probability of all realizations should add up to 1 and be positive.

$$\min_{\pi, Pr} \sum_{\psi \in \Psi} \omega_\psi * (f_\psi(\pi, Pr) - VAL_\psi)^2 \tag{3.21}$$

$$\sum Pr * M = 1 \tag{3.22}$$

$$Pr \geq 0 \tag{3.23}$$

In this paper, we consider the first 4 moments: mean, variance, skewness and kurtosis. A non-linear objective function allows to reset the initial values and execute the model until a good solution is obtained. We assume that the demand of final material are both period and product independent. Multi-product and multi-period scenario trees are generated in this paper. Three products, six time periods and four statistical properties lead to $|\Psi| = 72$ specified statistical properties. The minimum number of realizations in each period is four, and we choose to create five realizations in each period since we need to balance the trade-off between the quality of the solution and the complexity of the problem. The GAMS (General Algebraic Modeling System) is used to solve this non-linear optimization problem. Due to the fact that there can be multiple optimal solutions, we created four different scenario trees each has 5^6 scenarios in order to compare and validate the results. All the scenario trees have objective values of zero implying that the discrete realizations have a perfect match with the specified properties of continuous distribution, and satisfactory results are reached (Høyland and Wallace, 2001). It should be noted that we only include the realizations in the first period since the demand is period-independent.

3.3.2.2 Scenario reduction technique

Each scenario tree we generated has 5^6 scenarios and solving a NP-Hard problem with this amount of scenarios becomes computationally intractable. Therefore, we adopted scenario reduction to reduce the computational complexity. There are two types of scenario reduction techniques, one is called fast forward selection (FFS) and the other one is called backward selection (BS). The FFS outperforms the BS when the size of selected scenarios is no more than 25% of the size of original scenarios. FFS is used in this paper as sample size after reduction is approximately 1% of original scenario size. We decided to keep different scenario sample sizes and test the stability as well as the quality of scenario reduction.

S is a scenario set in which s = 1 $\cdots$ S. Each scenario can be represented by μ_s which has probability ν_s. L function measures the euclidean distance between two different scenarios. For each scenario, we calculate the overall weighted distance to the rest of scenarios, which is $WD_k^{[\eta]}$. $U^{[\eta]}$ contains all of the unselected scenario up to iteration η. It should be noted that when $\eta = 1$, all of the scenarios are unselected and Φ is empty. In detail, we measure the euclidean distance between each pair of scenario k and l where k,l $\in$ S. The overall weighted euclidean distance is stored in $WD_k^{[\eta]}$. Then we find the scenario with minimum overall distance and remove it from the unselected scenario set U. Next, distance matrix is updated because of the scenario we removed. Next scenario is selected using the same approach until enough scenarios are selected. After scenario selection, we assign the probability of those unselected scenario to the closest selected scenario. Feng and

Table 3.6: Notations for FFS

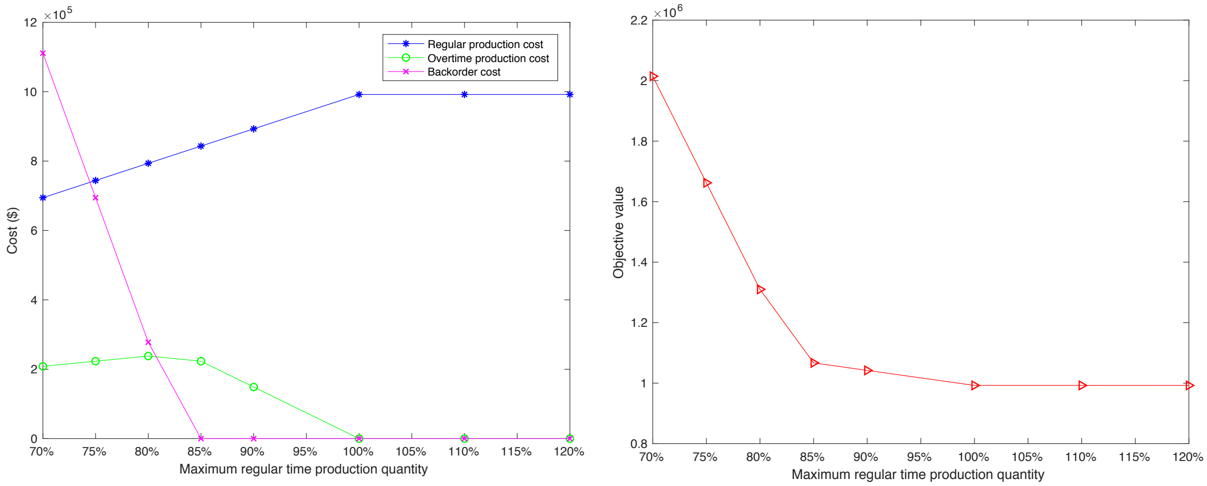
S	Scenario set
μ_s	Scenario s
ν_s	The probability of scenario s
$L(\cdot)$	Nonnegative function L_2-norm
$D_{k,l}^{[\eta]}$	Distance between scenario k and scenario l at iteration η
$WD_k^{[\eta]}$	Overall weighted distance of scenario k at iteration η
$U^{[\eta]}$	Set of unselected scenarios up to iteration η
Φ	Set of selected scenarios after reduction

Ryan (2013) studied five different sample sizes 10, 20, 30, 50, 100. We decide to keep 10, 15, 20, 30, 40, 80, 120 and 150 scenarios after reduction and reasons is two-fold: First, we want to test how scenario sample size affects the objective value. Second, we want to check if the results become stable as scenario sample size increases. One interesting observation is that for any i, j $\in$ {10, 15, 20, 30, 40, 80, 120, 150}, if we let λ_i indicates the scenario sample with size i, then $\lambda_i \subset \lambda_j \; \forall i < j$. For example, the scenario sample with size 30 is a proper subset of the scenario sample with size 40. This is due to that FFS algorithm is a construction process. Therefore, the sample under the larger size scenario is built upon the sample with the smaller size.

3.3.3 Analysis for the deterministic case

Results of the deterministic model are shown in Figures 3.2a and 3.2b. The objective value decreases as the maximum batch size increases because we have more regular time production resources. All of the production activity can be done in the regular time when the maximum batch size is 100% of the mean demand. Backorder exists when the maximum batch size is smaller than 85% of the mean demand. We have two different production capacities in this paper, which are the maximum batch size capacity and the maximum time capacity. When the maximum batch size is small, Equation 3.4 is the binding constraint and that is why objective value changes dramatically as we change $q_{i,t}$. When the maximum batch size is large enough, the overall cost becomes stable since the binding constraint becomes Equation 3.5, that is, we are running out of production time resource. Utilization of machine time ranges from 65% to 100% depending on the maximum batch size.

Different maximum batch sizes result in different production sequences, but those production sequences have similar setup cost, which means the maximum batch size only affects the production



(a) Various costs in the deterministic model

(b) Objective value in the deterministic model

Figure 3.2: Results of the deterministic model

schedule not the setup cost. One of the production sequences is shown in Table 3.7. It should be noted that the last setup in one period becomes as the first setup in the next period since we assumed that setup can be carried over from period to period. In order to save setup changeover cost, setups are typically saved and reused in the following time period . Note that the setup cost decreases when the maximum batch size changes from 110% to 120% indicating that extra products have been produced ahead of time in order to balance between inventory cost and setup cost. Carrying extra products increases inventory cost, but it can be justified with huge setup cost. In this case, producing and carrying extra products for future demand become beneficial.

Table 3.7: An optimal production sequence

Products	T_1	T_2	T_3	T_4	T_5	T_6
1	1	2	3	1	2	2
2	3	1	2	3	1	3
3	2	3	1	2	3	1

3.3.4 Analysis for the stochastic case

Four different scenario trees τ_1 to τ_4 were generated using moment matching technique as detailed in 3.3.2.1. The objective values equal to zero in the moment matching method indicating that those scenarios match the continuous distribution perfectly. Scenario reduction is used to select a subset which has a good representation of the original scenario set. In order to compare the solution for the two-stage model with the one for the multi-stage model, we need to find a reasonable scenario sample size. Solving the multi-stage stochastic programming problem to optimality is usually computationally intractable, so we decide to conduct stability test using the two-stage model and then compare the solution with the multi-stage model. The stability test aims to find a good scenario sample size such that the objective value is stable. Intuitively, when the scenario sample size is small, we only keep the scenarios in the central of the set and omit other scenarios that is far from center. On the other hand, as we increase the scenario sample size, the representativeness improves and problem becomes more complicated. Balancing the quantity of scenarios and computational complexity is an important step. The details of the relationship between sample size and objective value are included in Figure 3.3.

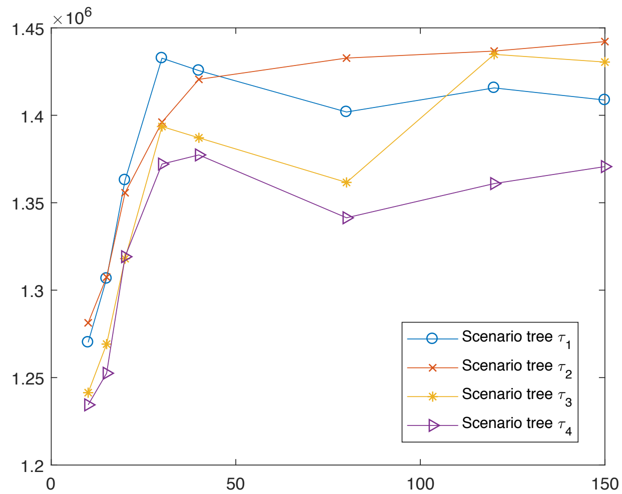


Figure 3.3: Scenario sample size stability test

The horizontal axis is the number of scenarios after reduction and the vertical axis is the objective values. At the beginning, the objective value has an increasing trend implying that scenario sample sizes like 10, 15, and 20 do not have a really good representation of the original distribution. For example, we may lose some extreme large values or extreme small values when the scenario sample size is small since the focus on is the centroid of the set. As the sample size increases, the mathematical model starts incorporating a more accurate representation of the continuous distribution. In conclusion, when the sample size is smaller than 30, all of scenario trees agree that sample size is insufficient due to increasing trend in the objective value. But when the sample size is bigger than 30, four scenario trees start having different behaviors since different scenario trees cover different aspects of the distribution. We decide to select 30 as the sample size. In addition, when we change the scenario sample size from 30 to 150, range of the objective value is really stable although the result of an individual tree changes randomly. The randomness comes from the fact that for a multi-period tree, one cannot simply compare solutions from different trees, as the nodes beyond the root do not coincidec(Heitsch and Römisch, 2003). The reason why variation, in Figure 3.3, increases as we enlarge the sample size is that four different scenario trees have similar representation of the centroid of the continuous distribution but other representations like variance, skewness, and kurtosis are slightly different. Hence, when scenario sample size is large, different scenario trees will have different realizations. Next, we conducted weak out-of-sample stability test which is defined as follows:

$$f(\Lambda_i, \tau_j) \approx f(\Lambda_j, \tau_i)$$

Λ_i includes all the baseline production decisions at root for scenario tree τ_i. The purpose of this test is to verify whether the scenario sample size we pick in the stability test is good enough. If the scenario tree is weak out-of-sample stable, we should get approximately the same optimal objective values when we solve one scenario tree with the root decisions fixed to the value we get from another tree. The details of weak out-of-sample stability test are included in the Table 3.8. The biggest difference can be found by applying the root decisions Λ_4 to the scenario tree τ_1, and vice versa. Since the gap is less than 5%, we claim that our scenario sample size stability test is valid.

Table 3.8: Weak out-of-sample stability test

	τ_1	τ_2	τ_3	τ_4
Λ_1	1,432,658	1,397,574	1,402,861	1,372,923
Λ_2	1,434,616	1,396,084	1,403,776	1,372,938
Λ_3	1,451,086	1,414,833	1,393,577	1,386,877
Λ_4	1,434,973	1,396,948	1,403,319	1,372,113

Comparison of different models can be conducted after obtaining the scenario sample size. Expected value of perfect information (EVPI) measures how much money the perfect information worths and value of stochastic solution (VSS) implies the difference between the deterministic model and stochastic model. Clearly, large EVPI and VSS indicate it is critical to consider uncertainty. EVPI_τ range from 195,684 to 211,513 which are approximately 15% of the two-stage stochastic objective value and VSS_τ range from 210,057 to 256,162 which are approximately 15% to 18% of the two-stage stochastic objective values. Significant EVPI and VSS clearly show that the two-stage stochastic programming model outperforms the deterministic model and considering the uncertainty is necessary. Value of multi-stage stochastic programming (VMS) and relative value of multi-stage stochastic programming (RVMS) are used to measure the improvement of solution by using multi-stage stochastic programming. Clearly, VMS and RVMS are non-negative since multi-stage stochastic program is a relaxation of the two-stage stochastic program.

$$VMS = RP^{TS} - RP^{MS} \qquad RVMS = \frac{RP^{TS} - RP^{MS}}{RP^{TS}}$$

RP^{TS} and RP^{MS} denote the optimal objective values of the two-stage model and of the multi-stage model.

However, multi-stage stochastic programming comes at the expense of solving a much larger and more difficult optimization model and obtains an optimal solution is computationally intractable, we consider the following lower bound:

$$VMS \geq RP^{TS} - RP^{MS}_F \qquad RVMS \geq \frac{RP^{TS} - RP^{MS}_F}{RP^{TS}}$$

Where RP_F^{MS} is a feasible solution to the multi-stage stochastic programming problem. The details of comparison can be found in Table 3.9. By applying the multi-stage stochastic programming in the field of semiconductor tool production, Huang and Ahmed (2009) reported their RVMS varies from less than 5% to around 70% depending on the setup. In order to make a fair comparison, we stop the models at a point where the computation times are around 24 hours and no big improvement in the optimality gaps. Our RVMS values are slightly larger than 10% meaning that, given 24 hours decision making time, we can improve our decision quality by more than 10%.

Table 3.9: Comparison of two-stage and multi-stage objective values

	RP^{TS}	RP_F^{MS}	VMS_{Lower}	RVMS_{Lower}	Optimality gap
τ_1	1,432,658	1,286,643	146,015	10.2%	0.71%
τ_2	1,396,084	1,254,663	141,421	10.1%	0.34%
τ_3	1,393,577	1,213,515	180,062	12.9%	1.56%
τ_4	1,372,113	1,225,938	146,175	10.7%	0.45%

Note that VMS measures the difference between RP^{TS} and RP^{MS}, and this value is also the difference between the expected value of perfect information to the two-stage model (EVPI^{TS}) and to the multi-stage model (EVPI^{MS}). EVPI measures how much money a decision maker is willing to pay for the perfect information in the future. A big EVPI means uncertainty is worth to be incorporated into the decision making process. In this case study, our VMS value is the amount of savings when perfect information becomes available, which is about 10% of the total cost. It should be noted that implementing the multi-stage stochastic programming model over the two-stage model is worthwhile due to 10% cost reduction and the 20-hr computation time is manageable for a production planning horizon of 6 months

3.4 Conclusions

This paper aims to design a multi-stage stochastic programming model to deal with demand uncertainty. A manufacturing plant in the automotive industry has been analyzed in the case study. Scenario generation and reduction techniques have been used to generate scenarios and reduce scenario sample size. Stability test was conducted to examine whether the scenario sample has good representation or not. Results of the two-stage stochastic programming model indicate the importance of considering uncertainty. Improvement in the objective value using multi-stage model has been analyzed.

Fast Forward Selection is used for scenario reduction to ensure good representation of the probabilistic distribution. Based on the sampling stability test, scenario size was kept at 30 after reduction. Compared the deterministic model with the two-stage stochastic programming model, EVPI and VSS are 15% and 18% of the objective value, respectively. It indicates the importance of considering uncertainty. VMS and RVMS are measured to compare the two-stage model with the multi-stage model. Significant VMS indicates big EVPI gap which means significant cost reduction when the multi-stage stochastic model is used for production planning. A big RVMS implies a non-negligible percentage difference in the objective value between the two-stage model and the multi-stage model. We calculated the lower bounds on VMS and RVMS due to the complexity of the multi-stage stochastic programming and thus optimality may not be able to achieve. Results show that the quality of solution can be improved by approximately 10% using the multi-stage stochastic model instead of the two-stage stochastic model.

In summary, this paper presents a multi-stage stochastic programming model to study the lot-sizing and scheduling problem under uncertainty. However, our research has following limitations. Firstly, demand is assumed to be product and period independent. This assumption can be invalid in reality. For example, demand of some automobile parts can heavily depends on the historical data. Secondly, multiple uncertain factors can be studied in our future works as we only focus on one of them. Thirdly, we subjectively determine that the result of scenario sample size is stable when the changing of the objective value is less than 5% which can be a big gap in some other fields. Lastly, heuristics can be designed due to high computational complexity of the multi-stage stochastic programming model. Those limitations should be addressed in our future research.

CHAPTER 4. A HYBRID STOCHASTIC AND ROBUST OPTIMIZATION MODEL FOR LOT-SIZING AND SCHEDULING PROBLEMS UNDER UNCERTAINTIES

Uncertainty is among the significant concerns in production scheduling. It has become increasingly important to take uncertainties into consideration for lot-sizing and scheduling. In this paper, we adopt the Hybrid Stochastic and Robust Optimization (HSRO) approach in lot-sizing and scheduling problems in which suppliers have the flexibility of satisfying a fraction of demand based on the market and their policies. Two types of uncertainties have been considered simultaneously: demand and overtime processing cost. Robust optimization is adopted for uncertain demand and Sample Average Approximation (SAA) technique is applied to solve the stochastic program for uncertain overtime processing cost. Numerical results based on a manufacturing company has been conducted to not only validate the proposed hybrid model but also quantitatively demonstrate the merit of our approach. Sample size stability test and sensitivity analyses on various parameters have also been conducted.

4.1 Introduction

Efficient and robust production planning is essential for manufacturing companies to stay competitive. Tomotani and de Mesquita (2018) pointed out that lot-sizing and scheduling are closely related that both decisions have to be made simultaneously to avoid sub-optimal results arising from considering them separately. As pointed out by Yang et al. (2017), the lot-sizing and scheduling decisions are challenging due to the various uncertainties including material shortage, machine breakdowns and demand fluctuation. In addition, replenishing inventory, seeking a new material supplier, purchasing and rearranging machines can be time consuming, so it is almost impossible to make timely response and adjustments to system oscillations (Fattahi et al., 2015; Pishvaee et al., 2011). Therefore, the design of lot-sizing and scheduling system must be robust to deal with uncertainties in the production processes.

Scenario-based stochastic programming approach has been gaining popularity in studying uncertainties in lot-sizing and scheduling problems. The probability distributions for the uncertain parameters are estimated and then scenarios are generated based on the modeling assumptions. Taş et al. (2018) considered stochastic setup time which follows a Gamma distribution and SAA technique was used to solve this stochastic programming problem. In addition, two heuristic algorithms were developed and evaluated in the paper. Hu and Hu (2016, 2018) studied a two-stage stochastic programming approach under demand uncertainty and later, extended to multi-stage stochastic programing model. The moment matching method was used to generate scenarios. Ramaraj et al. (2019) studied the similar production problem with demand and raw material quality uncertainties. However, this approach is only suitable for situations where the underlying probability distribution of the uncertain variable is known. The major drawbacks of this approach are: First, in some real-world applications, the decision makers may not have enough historical data to fit accurate probability distribution functions for the uncertain parameters. For example, it is almost impossible to predict future demand for a new product due to lack of historical data (Keyvanshokooh et al., 2016; Scarf, 1957). One good example is to predict future demand for the fashion industry. A lot of fashion products tend to be unique and hence there is not much historical data for fitting an appropriate distribution. Secondly, good approximation for continuous distributions requires a large number of scenarios. In a multi-period problem, overall scenario size grows exponentially with the number of scenarios in each time period. Although techniques like Fast Forward Selection (FFS) and Simultaneous Backward Reduction (SBR) have been used to reduce scenario sample size, important information may be lost during the process (Heitsch and Römisch, 2003). On the other hand, if scenario sample size is limited due to computational complexity, the accuracy of prediction for the future stage can be restricted and solutions may not be feasible for some extreme realizations of uncertainties. Finally, different decision makers may have different attitudes toward uncertainty. Scenario-based stochastic programming approach focuses on the expected value which assumes the decision makers care about the average performance of each scenario. However, in some cases, the decision makers can be more concerned about the worst case scenario than the average scenario.

To address these drawbacks, Robust Optimization (RO) has been utilized as an alternative technique to deal with uncertainties. Soyster (1973) assumed that all uncertain parameters would take their worst-case values within sets and this approach was perceived as overly conservative for practical implementation. In the mid-1990s, the shortcoming of over-conservatism was addressed by constrainting the uncertain parameters to belong to ellipsoidal uncertainty set. This approach only considers outcomes that are likely to happen but results in a non-linear robust counterpart (Ben-Tal and Nemirovski, 1998, 1999, 2000). More recently, Bertsimas and Sim (2003) proposed a new robust optimization approach that overcame the issue of high complexity when formulating the robust counterpart. Concretely, the robust counterpart of a linear programming problem is still a linear programming problem. The major advantages of using RO are: First, RO is not based on a probabilistic theory and does not requires extensive amount of historical data to support the parameter estimation. In other words, this approach does not need knowledge of specific probability distribution for the uncertain parameters. Second, this RO approach is computationally tractable because of linear robust counterpart. Curcio et al. (2018) considered adjustable robust optimization for lot-sizing and scheduling problem under multi-stage demand uncertainty. The results showed that their algorithm is much better than the deterministic model. Diaz et al. (2017) considered a production planning problem with uncertain operating and environment conditions. Single and multi-objective formulation for robust optimization were studied. Results show a significant correlation between robustness and sample size in the performance evaluation.

There are two types of uncertainties considered in this paper. Historical data is available and reliable to generate scenarios for overtime processing cost and hence we adopt the stochastic programming approach for this type of uncertainty. Due to the unpredictable characteristic, we adopt robust optimization to address demand uncertainty. The major challenge that many companies face is to predict accurate demand volume. If the demand volume is underestimated, customer demand may not be satisfied. On the other hand, if the demand volume is overestimated, then unnecessary inventory cost will incur. Overtime production is highly related to the demand prediction since it helps companies fulfill unpredicted demand in the peak season. The need to study the integration of these two types of uncertainties has been justified by Davis (1993). The author identified three major supply chain uncertainties, process uncertainty, and demand uncertainty. Supply uncertainty depends on the suppliers' reliability. Process uncertainty is related to the production process. Demand uncertainty often arises from inaccurate forecast and market fluctuation.

Govindan et al. (2017) pointed out that demand quantity, production and transportation costs are the most frequently studied uncertainties in supply chain. Li and Hu (2017) studied lot-sizing and scheduling problem under demand and workforce uncertainties. Ramaraj et al. (2017) considered the same production problem with demand and yield uncertainties. The difference is that they adopt stochastic programming approach for all uncertain parameters in the model. Alem et al. (2018) formulated stochastic and robust models separately and compared the results with Monte Carlo simulation. They provided guidelines for decision makers to assess a priori approach based on their preferences. Keyvanshokooh et al. (2016) used hybrid approach in the context of a closed loop supply chain problem where uncertain transportation cost was handled by stochastic programming approach and robust optimization was adopted to deal with demand uncertainty. Similar hybrid approach can be found in inventory control and bidding strategy (Liu et al., 2016; Minoux, 2018). In this paper, we adopt the HSRO approach introduced by Keyvanshokooh et al. (2016) in the context of lot-sizing and scheduling problems. To the best of authors' knowledge, no existing papers have studied the integration of demand and overtime processing cost uncertainties using HSRO approach in the application of lot-sizing and scheduling.

The major contributions of this study are listed as follows:

- A lot-sizing and scheduling framework is proposed to study the integration of multiple uncertainties. It provides the flexibility to satisfy only a fraction of customers according to market competition and company's policies.
- We adopt the HSRO approach of Keyvanshokooh et al. (2016) in a new context to study two different types of uncertainties simultaneously including stochastic programming approach for overtime processing cost uncertainty and robust optimization for demand uncertainty.
- SAA technique is applied to solve the stochastic program which considers overtime processing cost uncertainty.

The remainder of this paper is organized as follows: In section 4.2, we review the basic concept of robust optimization and present the proposed HSRO approach for lot-sizing and scheduling formulation. section 4.3 describes the computational results such as scenario stability test and sensitivity analyses on various parameters that provide managerial insights on the proposed model. Finally, section 4.4 summarizes the paper and suggests future research directions.

4.2 Problem description

In this paper, we consider a multi-product, multi-period, and capacitated lot-sizing and scheduling problem. Raw material are collected from up-stream factories and final products are shipped to customers and down-stream factories. There are multiple steps in the manufacturing process, such as welding, casting and molding. In this study, we assume that the products are perishable and hence the excess amount of inventory in one period cannot be carried over the used to fulfill demand in the future (Biller et al., 2005). There are several types of capacities such as maximum available time on the machine, maximum production batch size in both regular time and overtime. The goal is to design a robust production plan to maximize the profits given capacity constraints under demand and overtime processing cost uncertainties.

In the HSRO formulation, we introduce the penalty cost for unmet demand and surplus cost for excess production. On one extreme, if there is little market competition and company has significant market share, then losing small fraction of customers can be affordable. On the other extreme, if the market is very competitive then losing any customers may be unacceptable. The proposed hybrid model provides the flexibility to design an optimal plan under any circumstance between these two extreme situations. Penalty cost for unmet demand would be low if market is not competitive. On the other hand, penalty cost can be set to high values if customer satisfaction is critical. Introducing the penalty and surplus costs in the production balance the customer satisfaction and inventory resource requirement (Birge and Louveaux, 1997).

The objective of this study is to design a robust production plan under two different types of uncertainties: One for overtime processing cost and the other for customer demand. We assume that company has complete knowledge of the underlying probability distribution of the uncertain overtime processing cost, so stochastic programming can be used to model this type of uncertainty. On the other hand, predicting the probability density function of demand is very challenging due to several reasons. Demand could be influenced by unpredicted situations such as a competitor launches a new product or market fluctuation. Even if market is stable, predicting demand scenarios for new products is very difficult due to insufficient information, therefore, we adopt robust optimization to model demand uncertainty.

4.2.1 Mathematical notations

All notations for the mathematical formulation are listed in Table 4.1.

4.2.2 Robust optimization

We follow Bertsimas and Thiele (2006) to formulate the robust optimization component. Consider a linear programming problem where $C \in \mathbb{R}^n$, $b \in \mathbb{R}^m$ and A is a $m * n$ matrix.

$$\min \quad Cx \quad s.t. \quad Ax \leq b, \quad x \geq 0 \tag{4.1}$$

Without loss of generality, uncertainty is assumed to affect only the elements in the matrix A. We consider a particular row vector i of A and define J_i as the set of uncertain coefficients in that row. To simplify the exposition, every coefficient a_{ij}, j $\in J_i$ is subject to uncertainty and modeled as independent random variable which belongs to a symmetric interval $[\hat{a_{ij}} - \Delta a_{ij}, \hat{a_{ij}} + \Delta a_{ij}]$ where Δa_{ij} is the maximum deviation of the uncertain element a_{ij} and $\hat{a_{ij}}$ is the nominal value. This is reflected in the following formulation of the robust counterpart of Equation 4.1.

$$\min \quad Cx \quad s.t. \max_{\forall a_{ij} \in J_i} \left(\sum_j a_{ij} x_j \right) \leq b_i \quad \forall i, \ x \geq 0 \tag{4.2}$$

A scaled deviation parameter $z_{ij} = \frac{a_{ij} - \hat{a}_{ij}}{\Delta a_{ij}}$ is introduced. It should be noted that Δa_{ij}, $\hat{a_{ij}}$, a_{ij} represent the maximum deviation, nominal value, uncertain parameter for the random variable, respectively. A budget parameter Γ_i is used to setup a boundary for aggregated scaled deviation. This insight can be incorporated in mathematical terms as follows:

$$\sum_{j \in J_i} |z_{ij}| \leq \Gamma_i, \ \ \forall i \tag{4.3}$$

Table 4.1: Notations used in the HSRO formulation

Sets:		
i	$1, 2 \cdots, N$	Set of raw material comes from up-stream suppliers
j	$1, 2 \cdots, N$	i and j are alias
t	$1, 2 \cdots, T$	Set of time periods in the production horizon
s	$1, 2 \cdots, S$	Set of scenarios for overtime processing cost
Parameters:		
$D_{i,t}$	Stochastic demand for product i at time period t	
$D^s_{i,t}$	Uncertain demand for product i at time period t in scenario s	
$\hat{D^s_{i,t}}$	Nominal demand for product i at time period t in scenario s	
$\Delta D^s_{i,t}$	Maximum demand deviation from nominal value for product i at time period t in scenario s	
$\eta D^{s+}_{i,t}$	Positive deviation percentage from nominal demand for product i at time period t in scenario s	
$\eta D^{s-}_{i,t}$	Negative deviation percentage from nominal demand for product i at time period t in scenario s	
cap_t	Overall time availability on the machine at time period t	
c_i	Selling price for product i	
pt_i	Manufacturing time for product i	
pr_i	Regular time manufacturing cost for product i	
po_i	Stochastic overtime manufacturing cost for product i	
po^s_i	Overtime manufacturing cost for product i in scenario s	
$q_{i,t}$	The maximum regular time batch size for product i at time period t	
$sc_{i,j}$	Overall preparation cost when a setup changeover from two different products i, j is taken place	
$st_{i,j}$	Overall preparation time when a setup changeover from two different products i, j is taken place	
α	Ratio of regular and overtime production batch size	
N	Total amount of products that come from different families	
$prob_s$	The probability of scenario s	
pen	Penalty cost per unit of unmet demand	
sur	Surplus cost per unit of unnecessary production	
Γ^s_D	Overall demand deviation budget for scenario s	
Decision Variables:		
$X_{i,t}$	Production batch size in the regular time for product i at time period t	
$O^s_{i,t}$	Production batch size in the overtime production for product i at time period t in scenario s	
$Y_{i,j,t}$	Binary variable which takes value 1 if setup changeover from two different products i, j is taken place at time period t	
$Z_{i,t}$	Binary variable which takes value 1 if setup for product i is carried over from time period $t-1$ to t	
$V_{i,t}$	Production sequence at time period t. It takes value from 1 to N	

Based on the description above, set J_i is defined as $J_i = \{a_{ij} \big| a_{ij} = \hat{a_{ij}} + \Delta a_{ij} z_{ij}, \forall i, j, z \in \Psi\}$ where $\Psi = \{z \big| |z_{ij}| \leq 1,\ \sum_{j \in J_i} |z_{ij}| \leq \Gamma_i, \forall i\}$. Reformulating each constraint i as $\sum_j a_{ij} x_j = \sum_j (\hat{a_{ij}} + \Delta a_{ij} z_{ij}) x_j = \sum_j \hat{a_{ij}} x_j + \sum_j \Delta a_{ij} z_{ij} x_j$, the bilevel robust Equation 4.2 can be transformed into:

$$\min \ Cx \quad s.t. \ \sum_{j \in J_j} \hat{a_{ij}} x_j + \max_{z_i \in \Psi_i} \sum_{j \in J_i} \Delta a_{ij} z_{ij} x_j \leq b_i \ \ \forall i, \ \ x \geq 0 \tag{4.4}$$

The lower level problem $\max_{z_i \in \Psi_i} \sum_{j \in J_i} \Delta a_{ij} z_{ij} x_j$ for a given x^* and constraint index i is equivalent to Equation 4.5:

$$\max \sum_{j \in J_i} \Delta a_{ij} z_{ij} x_j^* \quad s.t. \ \sum_{j \in J_i} z_{ij} \leq \Gamma_i \ \ 0 \leq z_{ij} \leq 1 \ \ \forall j \in J_i \tag{4.5}$$

By introducing the dual variables λ_i and μ_{ij}, the dual of Equation 4.5 can be expressed as:

$$\min \ \Gamma_i \lambda_i + \sum_{j \in J_i} \mu_{ij} \quad s.t. \ \lambda_i + \mu_{ij} \geq \Delta a_{ij} x_j^*, \mu_{ij}, \lambda_i \geq 0 \quad \forall i, J_i \tag{4.6}$$

By applying the dual Equation 4.6 to Equation 4.4, the robust counterpart of Equation 4.1 is obtained:

$$\begin{aligned} & \min \ Cx \\ s.t. \quad & \sum_{j \in J_i} \hat{a_{ij}} x_j - \Gamma_i \lambda_i - \sum_{j \in J_i} \mu_{ij} \leq b_i \ \ \forall i \\ & \lambda_i + \mu_{ij} \geq \Delta a_{ij} x_j \ \ \forall i, j \in J_i \\ & \mu_{ij}, \lambda_i \geq 0 \ \ \forall i, j \in J_i \end{aligned} \tag{4.7}$$

For a given solution x_j^*, the probability of constraint violation can be calculated by (Bertsimas and Sim, 2004):

$$Pr\left(\sum_{j \in J_i} a_{ij} x_j^* > b_i \right) \leq 1 - \Phi\left(\left(\Gamma_i - 1 \right) / \sqrt{|J_i|} \right) \tag{4.8}$$

Reversely, we can setup a maximum violation probability ϵ_i and Equation 4.9 gives us the minimum budget Γ_i to maintain that particular level of violation probability.

$$\Gamma_i \geq 1 - \Phi^{-1}(1 - \epsilon_i) \sqrt{|J_i|} \tag{4.9}$$

4.2.3 Hybrid stochastic and robust optimization model

To introduce our hybrid model, we first introduce a two-stage stochastic programming model which serves as a baseline and the robust optimization is then built on top of the two-stage stochastic programming model to construct the full hybrid model.

4.2.3.1 Baseline model with uncertainty in overtime processing costs

In the two-stage stochastic programming model, demands are set to their nominal values while uncertainties in the overtime processing costs are incorporated with the stochastic programming approach. Sample Average Approximate approach was adopted to generate scenarios. In the lot-sizing and scheduling process, first-stage decision variables identify the baseline production i,e. raw material purchase and regular time production planning while second-stage decision variables define possible recourse like overtime production and compensatory actions (Alfieri et al., 2012). The first-stage decisions have to be made in the presence of uncertainties meaning the regular time production decisions are made before we observe the realization of uncertain overtime processing cost (Chaharsooghi et al., 2011). In the objective function, the goal is to maximize profit based on revenue and production costs. Production costs include regular time production cost, overtime production cost, and setup cost.

$$\begin{aligned}\max \zeta = &\sum_{i=1}^{N}\sum_{t=1}^{T} c_i * X_{i,t} + \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{S} prob_s * O_{i,t}^s * c_i - \sum_{i=1}^{N}\sum_{t=1}^{T} pr_i * X_{i,t} \\ &- \sum_{i=1}^{N}\sum_{i\neq j}\sum_{j=1}^{N}\sum_{t=1}^{T} sc_{i,j} * Y_{i,j,t} - \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{S} prob_s * po_i^s * O_{i,t}^s\end{aligned} \tag{4.10}$$

$$X_{i,t} + O_{i,t}^s = \hat{D_{i,t}^s} \qquad \forall i, t, s \tag{4.11}$$

Equation 4.11 ensure that overall production equals to the nominal demand. Regular time production $X_{i,t}$ is first-stage decision since it has to be determined before we realize the uncertainty. Overtime production is scenario-based second stage decision since they are served as compensatory actions. In addition, we assume that the products are perishable meaning excess inventory cannot be carried over to satisfy future demand. Backlog is not allowed and hence unmet demand will be lost. In the hybrid model, demands are random, which can vary in an predetermined interval with mean $\hat{D_{i,t}^s}$ and standard deviation $\Delta D_{i,t}^s$. Therefore, the overall production can be either greater than or less than the actual realization of demands. This will be discussed in detail in the hybrid model section.

$$X_{i,t} \leq q_{i,t} * (Z_{i,t} + \sum_{j\neq i}^{N} Y_{j,i,t}) \qquad \forall i, t \tag{4.12}$$

Equation 4.12 ensures that regular production quantity $X_{i,t}$ cannot exceed regular time production batch size capacity $q_{i,t}$. Since the setup of a particular product can take place at most once in each time period meaning $Z_{i,t} + \sum_{j\neq i}^{N} Y_{j,i,t} \leq 1$. For example, if the setup of product i_1 is carried over from t_1 to t_2, then $Z_{i_1,t_2} = 1$. Furthermore, we know that $\sum_{j\neq i_1}^{N} Y_{j,i_1,t_2} = 0$ based on the setup assumption. On the other hand, for any product $i_2 \neq i_1$, $Z_{i_2,t_2} = 0$ since the carried over setup in time period t_2 is not i_2. It provides the opportunity to setup product i_2 during this time period. If product i_2 is scheduled to be manufactured after product i_1, then we know that $Y_{i_1,i_2,t_2} = 1$.

$$\sum_{i=1}^{N} pt_i * X_{i,t} + \sum_{i=1}^{N}\sum_{i\neq j}\sum_{j=1}^{N} st_{i,j} * Y_{i,j,t} \leq cap_t \qquad \forall t \tag{4.13}$$

Equation 4.13 states the overall regular time capacity. The left side represents the total production time includes regular time processing time and setup changeover time. It is assumed that setup is only required when products change from different families.

$$O_{i,t}^{s} \leq \alpha * X_{i,t} \qquad \forall i, t, s \tag{4.14}$$

In Equation 4.14, overtime production quantity is bounded by the regular time production quantities. Government policy states that the overtime production batch size must be limited to 20% of the regular production batch size. Although overtime exceeds that threshold can be considered under extreme circumstances, but it could lead to injury, fatigue and reduce efficiency (Zhang et al., 2011).

$$\sum_{i=1}^{N} Z_{i,t} = 1 \qquad \forall t \tag{4.15}$$

$$X_{i,t} = 0 \qquad \forall i, t = T \tag{4.16}$$

Equation 4.15 indicates that only one setup can be carried over to the following time period. Equation 4.16 states that no regular time production activity is allowed in the dummy period ($t = T$).

$$Z_{i,t} + \sum_{j \neq i}^{N} Y_{j,i,t} = Z_{i,t+1} + \sum_{j \neq i}^{N} Y_{i,j,t} \qquad \forall i, t = 1 \cdots T-1 \tag{4.17}$$

Equation 4.17 indicates that setups have to happen in an equilibrium state. One of assumptions is that setups are preservable meaning the last setup in one time period can be carried over to the next period. For example, consider a situation where only products i_1 are manufactured in time period t_1 and the setup carried over from t_0 is i_1, then $Z_{i_1,t_1} = Z_{i_1,t_2} = 1$, since the setup for product i_1 is carried over from t_1 to t_2. In addition, there is no setup changeover either from other products to i_1 or from i_1 to any other products. Therefore, $\sum_{j \neq i}^{N} Y_{j,i,t} = \sum_{j \neq i}^{N} Y_{i,j,t} = 0$ and equality condition is met. Consider a more complicated case where three different products are manufactured in t_1 in sequence of $i_1 - i_2 - i_3$ and the setup carried over from previous time period is i_1. We know $Z_{i_1,t_1} = Z_{i_3,t_2} = 1$ since i_3 is the last product on the production line in time period t_1. In addition, $Y_{i_1,i_2,t_1} = Y_{i_2,i_3,t_1} = 1$ since two setup changeovers take place in time period t_1. In conclusion, Setups carried over from previous time period and changeover from other products represent flows going into a product node while setups carried over to the next time period and changeover to other products represent flows leaving a product node.

$$V_{j,t} \geq V_{i,t} + 1 - N * (1 - Y_{i,j,t}) \qquad \forall i, j \neq i, t \tag{4.18}$$

One important assumption is that the setup for a particular product can take place at most once in each time period. Equation 4.18 is a sub-tour elimination constraint. N is the number of products and $V_{i,t}$ is the production sequence of product i in time period t. We explain how this type of constraints avoid sub-tour with an example in Figure 4.1. Assuming there are five different products to be manufactured in time period t and production sequence is $1-2-3-4-5$. Then $N = 5$, $V_{1,t} = 1$, $V_{2,t} = 2$, $V_{3,t} = 3$, $V_{4,t} = 4$ and $V_{5,t} = 5$. Let's introduce an extra node 0, and without loss of generality, we assume all feasible paths start from node 0 and end at node 0.

Consider there is a sub-tour skipping node 0 and going directly from node 5 back to node 1, then there is a sub-tour. This situation is represented by the dash line in the figure. Since this is a 5-step sub-tour, if we sum up all $Y_{i,j,t} = 1$, we will have $5 \leq 4$ which is impossible. If all feasible paths have to go through node 0, then there is no sub-tour in the graph. If product i is followed by product j, then we know $Y_{i,j,t} = 1$ and $V_{j,t} = V_{i,t} + 1$. Otherwise, we have $V_{j,t} - V_{i,t} \geq 1 - N$. Both inequalities are valid since, for decision variables V, the minimum value is 1 and maximum value is N.

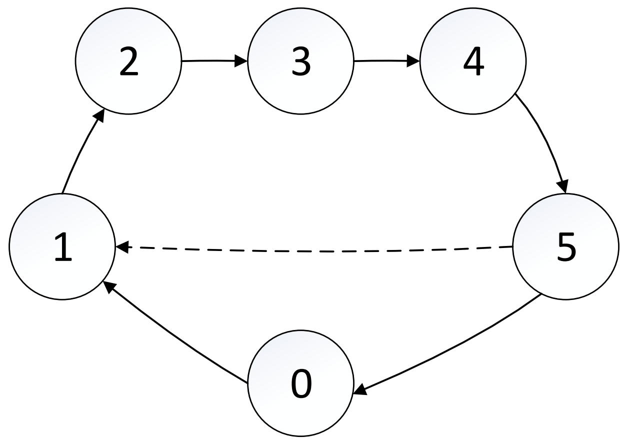


Figure 4.1: Sub-tour elimination example

4.2.3.2 Hybrid model with uncertainties in overtime processing costs and demand

In this paper, we adopt the HSRO approach proposed by Keyvanshokooh et al. (2016) in the lot-sizing and scheduling problem setting. SAA technique is applied to stochastic overtime processing cost. SAA is a Monte Carlo simulation based approach and scenarios are generated by taking random IID samples from the given distribution (Kleywegt et al., 2002). Within each scenario, we define polyhedral uncertainty sets for demand in each time period and for each product. The details are shown in Figure 4.2. There are $|S|$ different realizations of stochastic overtime processing cost. Inside each scenario, we define a symmetric interval for robust demand. The nominal demand $\hat{D^s_{i,t}}$ and maximum deviation $\Delta D^s_{i,t}$ are predetermined. Uncertain demand $D^s_{i,t}$ is allowed to deviate from the $\hat{D^s_{i,t}}$ toward the worst-case value within that interval. Note that strips in Figure 4.2 are symmetric with respect to the nominal value, but the width of the strips do not need to be the same for different time periods or products.

To develop the uncertainty sets for demand, we introduce the positive and negative deviation percentages from their nominal demands as follows:

$$\eta D^{s+}_{i,t} = \frac{D^s_{i,t} - \hat{D^s_{i,t}}}{\Delta D^s_{i,t}} \quad \text{if} \quad D^s_{i,t} \geq \hat{D^s_{i,t}} \qquad \eta D^{s-}_{i,t} = \frac{\hat{D^s_{i,t}} - D^s_{i,t}}{\Delta D^s_{i,t}} \quad \text{if} \quad D^s_{i,t} \leq \hat{D^s_{i,t}} \tag{4.19}$$

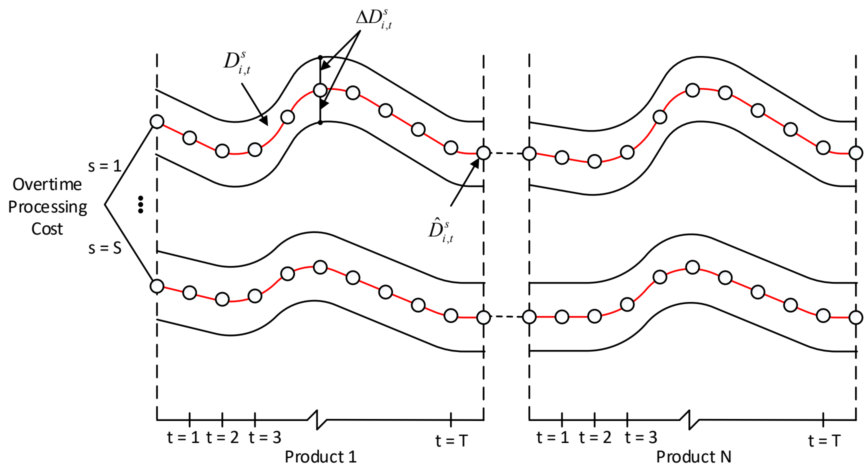


Figure 4.2: Characterization of uncertain demand and overtime processing cost

Notice that at most one of $\eta D^{s+}_{i,t}$ and $\eta D^{s-}_{i,t}$ will be positive and the other one has to be zero in Equation 4.19. The polyhedral uncertainty sets of demand for each scenario of overtime processing costs can be represented by:

$$J_D^S = \left\{ D^s_{i,t} \middle| D^s_{i,t} = \hat{D^s_{i,t}} + \Delta D^s_{i,t} * \eta D^{s+}_{i,t} - \Delta D^s_{i,t} * \eta D^{s-}_{i,t} \ \ \forall i,t \ \forall \eta D^{s+}_{i,t}, \eta D^{s-}_{i,t} \in K_D \right\} \tag{4.20}$$

where

$$K_D = \left\{ \eta D^{s+}_{i,t}, \eta D^{s-}_{i,t} \middle| 0 \le \eta D^{s+}_{i,t} \le 1, \ \ 0 \le \eta D^{s-}_{i,t} \le 1, \ \sum_{i,t} (\eta D^{s+}_{i,t} + \eta D^{s-}_{i,t}) \le \Gamma^s_D \right\} \tag{4.21}$$

In particular, if a realization of demand is above the mean value $\hat{D_{ij}}$, then $\eta D^{s+}_{i,t}$ is strictly positive and $\eta D^{s-}_{i,t}$ equals to 0. Corresponding uncertainty sets of demand becomes $J_D^S = \left\{ D^s_{i,t} \middle| D^s_{i,t} = \hat{D^s_{i,t}} + \Delta D^s_{i,t} * \eta D^{s+}_{i,t} \right\}$. Reversely, if a realization of demand is below the mean demand $\hat{D_{ij}}$, then $\eta D^{s-}_{i,t}$ is strictly positive and $\eta D^{s+}_{i,t}$ equals to 0. Corresponding uncertainty sets of demand becomes $J_D^S = \left\{ D^s_{i,t} \middle| D^s_{i,t} = \hat{D^s_{i,t}} - \Delta D^s_{i,t} * \eta D^{s-}_{i,t} \right\}$. For a given overtime processing cost scenario, the dimension of those sets is $|i| * |t|$, which means the budget for a given scenario (Γ^s_D) can take any value between 0 to $|i| * |t|$. If Γ^s_D equals to 0, then there is no protection against uncertain demand and our hybrid model is identical to the two-stage model. If Γ^s_D equals to $|i| * |t|$, then corresponding constraint is fully protected.

In the HSRO formulation, demands belong to some predefined intervals and the exact values are not known. If we include Equation 4.11 in the model formulation, then there is no guarantee that these constraints will be satisfied because the exact realization of uncertain parameter is unknown before measuring the optimal regular and overtime production quantities. On the other hand, solving the problem with Equation 4.11 will end up with two results: (1) optimal production quantity is larger than the demand or (2) optimal production quantity is smaller than the demand. In both cases, constraint Equation 4.11 is not satisfied and the decision making becomes infeasible.

In order to avoid this situation, we decide to move this constraint to the objective function with new cost parameters. Penalty cost *pen* is introduced for one unit of unsatisfied demand and surplus cost *sur* is introduced for one unit of excess production over demand. These two new cost parameters provide the flexibility for the decision makers based on the market environment and policy. For example, if the products have large market share and there is not much competition, then the company can afford unmet demand and satisfy only proportion of orders. In this case, the decision maker can decrease penalty cost and increase surplus cost. Conversely, if the company is in a very competitive market and losing a customer incurs a high cost, then we should satisfy as much demand as possible since losing customers become expensive. In this case, we can increase penalty cost and decrease surplus cost. Our goal is to minimize the maximum amount of violation cost due to unbalanced production flow in Equation 4.11. In the HSRO formulation, this set of constraint is incorporated into the objective function with penalty cost *pen* and surplus cost *sur*. For a given overtime processing cost scenario s, the violation cost can be formulated as follows:

$$VC_s(X,O) = \max_{D^s_{i,t} \in J^S_D} \left\{ \sum_{i,t} \left(D^s_{i,t} - X_{i,t} - O^s_{i,t} \right) * pen, \sum_{i,t} \left(X_{i,t} + O^s_{i,t} - D^s_{i,t} \right) * sur \right\} \tag{4.22}$$

where $D^s_{i,t}$ is the random demand which belongs to the sets Equation 4.20 and 4.21. In Equation 4.22, $X_{i,t}$ is the regular time production decision and $O^s_{i,t}$ is the overtime production decisions. The first term in the violation cost $\sum_{i,t} \left(D^s_{i,t} - X_{i,t} - O^s_{i,t} \right)$ is unmet demand and the second term in the violation cost $\sum_{i,t} \left(X_{i,t} + O^s_{i,t} - D^s_{i,t} \right)$ is excess production over demand. For each overtime processing cost scenario, Equation 4.22 is the worst-case result for violation cost and we want to minimize this violation cost. By introducing an auxiliary variable W_s for each overtime processing cost scenario, we transform the non-linear programming formulation to a linear programming formulation:

$$\begin{aligned} \min \;& VC_s(X,O) = W_s \\ s.t. \;& \sum_{i,t} \left(D^s_{i,t} - X_{i,t} - O^s_{i,t} \right) * pen \le W_s, \;\; \forall D^s_{i,t} \in J^S_D \\ & \sum_{i,t} \left(X_{i,t} + O^s_{i,t} - D^s_{i,t} \right) * sur \le W_s, \;\; \forall D^s_{i,t} \in J^S_D \\ & W_s \ge 0 \end{aligned} \tag{4.23}$$

For a given overtime processing cost scenario s, Equation 4.23 should always be feasible for any realizations of uncertain demand within their polyhedral sets. Then we find robust counterparts for each constraint in Equation 4.23:

$$\max_{D^s_{i,t} \in J^S_D} \left\{ \sum_{i,t} \left(D^s_{i,t} - X_{i,t} - O^s_{i,t} \right) * pen \right\} \le W_s, \tag{4.24}$$

which can be rewritten as:

$$\begin{aligned} \max_{\eta D^{s+}_{i,t}, \eta D^{s-}_{i,t} \in K^d} & \left\{ \sum_{i,t} \left(\Delta D^s_{i,t} * \eta D^{s+}_{i,t} - \Delta D^s_{i,t} * \eta D^{s-}_{i,t} \right) * pen \right\} \\ & + \sum_{i,t} \left(\hat{D^s_{i,t}} - X_{i,t} - O^s_{i,t} \right) * pen \leq W_s \end{aligned} \tag{4.25}$$

We optimize Equation 4.25 over the positive $(\eta D^{s+}_{i,t})$ and negative $(\eta D^{s-}_{i,t})$ percentage of deviation from the nominal value. Let's focus on the first term in the Equation 4.25. It should be noted that the differences between the first term in Equation 4.25 and Equation 4.26 are: (1) maximization objective function has been changed to a minimization objective function; (2) we add a negative sign in front of the coefficients in the objective function.

$$\begin{aligned} \min_{\eta D^{s+}_{i,t}, \eta D^{s-}_{i,t} \in K^d} & \left\{ \sum_{i,t} \left(-\Delta D^s_{i,t} * \eta D^{s+}_{i,t} + \Delta D^s_{i,t} * \eta D^{s-}_{i,t} \right) * pen \right\} \\ s.t. \quad & 0 \leq \eta D^{s+}_{i,t} \leq 1 \\ & 0 \leq \eta D^{s-}_{i,t} \leq 1 \\ & \sum_{i,t} \left(\eta D^{s+}_{i,t} + \eta D^{s-}_{i,t} \right) \leq \Gamma^s_D \end{aligned} \tag{4.26}$$

Notice that the domains of $\eta D^{s+}_{i,t}$ and $\eta D^{s-}_{i,t}$ stay the same, therefore, the objective value of Equation 4.26 equals to the negative objective value of the first term in Equation 4.25. We add an additional negative sign in front of the minimization function in Equation 4.27 such that it is equivalent to the first term in Equation 4.25. $\alpha 1^s_{i,t}, \alpha 2^s_{i,t}$ and $\beta 1^s$ are the dual variables for constraints that have been transformed into the standard form in Equation 4.27.

$$\begin{aligned} - \min_{\eta D^{s+}_{i,t}, \eta D^{s-}_{i,t} \in K^d} & \left\{ \sum_{i,t} \left(-\Delta D^s_{i,t} * \eta D^{s+}_{i,t} + \Delta D^s_{i,t} * \eta D^{s-}_{i,t} \right) * pen \right\} \\ s.t. \quad & -\eta D^{s+}_{i,t} \geq -1 \ \forall i,t \qquad \alpha 1^s_{i,t} \\ & -\eta D^{s-}_{i,t} \geq -1 \ \forall i,t \qquad \alpha 2^s_{i,t} \\ & \sum_{i,t} \left(-\eta D^{s+}_{i,t} - \eta D^{s-}_{i,t} \right) \geq -\Gamma^s_D \qquad \beta 1^s \\ & \eta D^{s+}_{i,t}, \eta D^{s-}_{i,t} \geq 0 \end{aligned} \tag{4.27}$$

Then we take the dual of formulation Equation 4.27:

$$\begin{aligned} \max_{\alpha 1^s_{i,t}, \alpha 2^s_{i,t}, \beta 1^s} & \left\{ -\Gamma^s_D * \beta 1^s - \sum_{i,t} \left(\alpha 1^s_{i,t} + \alpha 2^s_{i,t} \right) \right\} \\ s.t. \quad & -\alpha 1^s_{i,t} - \beta 1^s \leq -\Delta D^s_{i,t} \qquad \forall i,t \\ & -\alpha 2^s_{i,t} - \beta 1^s \leq \Delta D^s_{i,t} \qquad \forall i,t \\ & \alpha 1^s_{i,t}, \alpha 2^s_{i,t}, \beta 1^s \geq 0 \qquad \forall i,t \end{aligned} \tag{4.28}$$

In the linear programming Equation 4.28, all decision variables are non-negative and $\Delta D^s_{i,t}$ is the maximum deviation from the nominal value which is also a positive constant. Therefore, $-\alpha 2^s_{i,t} - \beta 1^s \leq \Delta D^s_{i,t}$ becomes redundant since it can be satisfied in any condition. Due to the strong duality theory, we substitute the objective function of Equation 4.28 without $\alpha 2^s_{i,t}$ in the Equation 4.25. The robust counterpart of first constraint in Equation 4.23 can be expressed as follows:

$$\begin{aligned}
&\left\{ \sum_{i,t} \left(\hat{D^s_{i,t}} - X_{i,t} - O^s_{i,t} + \alpha 1^s_{i,t} \right) + \Gamma^s_D * \beta 1^s \right\} * pen \leq W_s \\
&\alpha 1^s_{i,t} + \beta 1^s \geq \Delta D^s_{i,t} \qquad \forall i,t \\
&\alpha 1^s_{i,t}, \beta 1^s \geq 0 \qquad \forall i,t
\end{aligned} \tag{4.29}$$

Applying the same procedure to the second constraint in Equation 4.23, the other robust counterpart is obtained as follows:

$$\begin{aligned}
&\left\{ \sum_{i,t} \left(X_{i,t} + O^s_{i,t} - \hat{D^s_{i,t}} + \alpha 2^s_{i,t} \right) + \Gamma^s_D * \beta 2^s \right\} * sur \leq W_s \\
&\alpha 2^s_{i,t} + \beta 2^s \geq \Delta D^s_{i,t} \qquad \forall i,t \\
&\alpha 2^s_{i,t}, \beta 2^s \geq 0 \qquad \forall i,t
\end{aligned} \tag{4.30}$$

Finally, our HSRO formulation of the lot-sizing and scheduling design problem becomes:

$$\begin{aligned}
\max \zeta = &\sum_{i=1}^{N}\sum_{t=1}^{T} c_i * X_{i,t} + \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{S} prob_s * O^s_{i,t} * c_i - \sum_{i=1}^{N}\sum_{t=1}^{T} pr_i * X_{i,t} \\
&- \sum_{i=1}^{N} \sum_{i \neq j} \sum_{j=1}^{N}\sum_{t=1}^{T} sc_{i,j} * Y_{i,j,t} - \sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{s=1}^{S} prob_s * po^s_i * O^s_{i,t} \\
&- \sum_{s=1}^{S} prob_s * W_s
\end{aligned} \tag{4.31}$$

subject to Equation 4.12 - 4.18, Equation 4.29 and Equation 4.30. For each overtime processing cost scenario in the hybrid model, the budget parameter Γ^s_D balances the trade-off between the level of robustness and the degree of conservativeness of the solution. As a consequence, larger Γ^s_D provides more protection and increases the level of robustness while smaller Γ^s_D results in higher probability of constraint violation. On the other hand, bigger Γ^s_D leads to lower expected profit since model allows more deviations toward the worst-case in their uncertainty sets.

4.3 Case study

In order to demonstrate and validate the proposed hybrid model, we conduct a case study on braking equipment production. A manufacturing plant receives two different types of raw material from upstream and produces three different types of braking actuators P_1, P_2, and P_3. These products are directly supplied to customers and the goal is to identify the optimal production strategy such that the total system cost is minimized.

4.3.1 Date sources

This case study considers three different final products ($N = 3$). According to government policy, ratio of overtime and regular time production batch sizes is set to 20% ($\alpha = 20\%$) meaning overtime production batch size can not bigger than 20% of regular time production batch size (Menezes et al., 2011). Setup changeovers are product-dependent and hence it is important to identify the optimal production sequence. Setup changeovers times are included in Table 4.2 and corresponding setup changeover costs are proportional to their setup changeover times (James and Almada-Lobo, 2011). Notice that setup time between products in the same families is zero and setup changeover matrix is not symmetric due to the fact that setups are product-dependent.

Table 4.2: Setup changeover times $st_{i,j}$ (mins)

	P_1	P_2	P_3
P_1	0	270	90
P_2	180	0	270
P_3	90	180	0

Table 4.3 provides the nominal demand, processing time, and regular time processing cost. Nominal demands do not change over the planning horizon and the maximum demand deviations can vary from 5% to 30% of their nominal values. Overtime processing cost follows a normal distribution with mean equals to 1.5 times regular time processing cost and standard deviation equals to 10% of mean value(Rego and Mesquita, 2015). Unit selling price, penalty cost, and surplus cost are proportional to regular time processing cost and more details on overall time capacity cap_t and maximum regular time batch size $q_{i,t}$ can be found in the Hu and Hu (2016).

Table 4.3: Parameters setup for $\hat{D^s_{i,t}}$, pt_i, and pr_i

	P_1	P_2	P_3
Nominal demand $\hat{D^s_{i,t}}$ (unit)	467.25	33.82	149.7
Processing time pt_i (mins/unit)	6	6.6	7.2
Regular time processing cost pr_i (\$/unit)	254.08	254.08	254.08

4.3.2 Computational results

In this section, we first present scenario stability test to validate that the scenario sample size is sufficient to generate stable objective function. The results are illustrated in Figure 4.3. Five different scenario sample sizes with 20 replications were analyzed. The key idea is that when several scenario samples with the same sample size are generated, the optimal value of objective function should be close if scenario sample size is sufficient enough to represent the distribution. Concretely, we generate 20 different scenario trees η_i $\forall i = 1, \cdots, 20$ and sample sizes are identical across all scenario trees. Suppose we solve the model with each scenario trees generated, the optimal solutions are x_i^* $\forall i = 1, \cdots, 20$ and optimal objective values are $f(x_i^*, \eta_i)$ $\forall i = 1, \cdots, 20$. Stability indicates that $f(x_i^*, \eta_i) \approx f(x_j^*, \eta_j)$ $\forall i, j \in \{1, \cdots, 20\}$ and $i \neq j$. If this type of stability is obtained, then the performance of optimal solution x^* and $f(x^*, \eta)$ are independent of which scenario tree gets selected Kaut et al. (2007). Scenario sample sizes = 20, 50, 80, 100, 150 have been tested and illustrated in Figure 4.3. Intuitively, when the scenario sample size is small, not enough information was collected to generate stable objective values. As scenario sample size increases, more information about the distribution becomes available and objective values become more stable. It can be observed from the figure that when scenario sample size is 20, the optimal objective values are highly variable and hence not stable. When scenario sample size is 150, the minimum objective value is 1,940,176 while the maximum value is 1,945,427. The lack of significant difference between different trials shows sample stability. The model statistics as a function of number of scenarios are shown in Table 4.4. The computational times are less than 1 minute and the size of problem increases linearly as a function of number of scenarios. From the CPU times, we can see that the number of scenarios does not increase the computational complexity dramatically. In order to investigate the factors that drive the major complexity of the model, the number of time periods and the number of products have been analyzed. The computational time increases linearly as a function of number of time periods. However, if we increase the number of products, the computational time increases much faster than other factors.

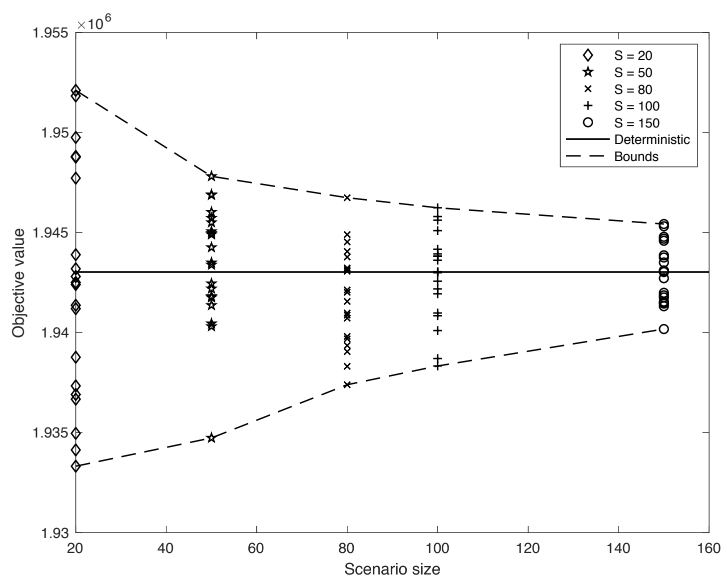


Figure 4.3: Scenario stability test

The solid line in Figure 4.3 represents the objective value of the deterministic model in which demand and overtime processing cost are fixed and known. It should be noted that roughly half of the objective values of the stochastic models are above that red line when the rest are below. The main reason is that we consider overtime processing cost uncertainty in the two-stage stochastic programming model and demands are assumed to be fixed at their nominal values. Therefore, uncertainty only affects the objective function of the two-stage stochastic programming model and feasible region stays the same. Concretely, if the scenarios have overtime processing cost higher than the nominal values, then the two-stage stochastic programming model will have higher objective value than the deterministic model. Reversely, if the scenarios have lower overtime processing

Table 4.4: Model statistics

Number of scenarios	Number of equations	Number of variables
20	1399	1426
50	3349	3406
80	5299	5386
100	6599	6706
150	9849	10006

cost than the nominal values, then the two-stage stochastic programming model will have lower objective value than the deterministic model. From the 20 random samples in Figure 4.3, roughly half of the objective values of the two-stage stochastic programming model are higher than the objective value of the deterministic model.

The comparison between the deterministic model, two-stage stochastic programming model and hybrid model is shown in Figure 4.4. The solid line represents the objective value of the deterministic model. When the budget $\Gamma_D^s = 0$, the objective value of the hybrid model equals to the objective value of the two-stage stochastic model since no demand uncertainty is allowed. Concretely, the top left point represents the objective value of the two-stage stochastic programming model. However, when we increases the budget, the objective value of the hybrid model decreases dramatically as shown in Figure 4.4. The key take-away are: First, the objective value of the deterministic model can be either higher or lower than the objective value of the two-stage stochastic programming model depending on the overtime production cost scenarios. Second, as budget increases, the objective value of the hybrid model will be significantly lower than the objective value of the deterministic model which concludes that considering uncertainties are really important.

We first perform the sensitivity analysis on parameters related to budget and quantity. The effect of budget uncertainty is studied by varying Γ_D^s for uncertain demands. Let's define ρ as the maximum variability of the uncertain demands. Higher ρ results in larger deviation. Note that budget can take any values between 0 to $|N| * |T| = 18$. If the budget happens to be integer, then it is the maximum amount of parameters that can deviate from their mean values. $\Gamma_D^s = 0$ indicates that there is no protection against uncertainty and $\Gamma_D^s = 18$ provides fully protection at expense of getting conservative solutions. For a particular trajectory in Figure 4.4, we fix ρ and vary Γ_D^s. As budget increases, we allow more variability in the uncertain demand and hence optimal profit becomes lower. One special case is $\Gamma_D^s = 0$, four different ρ values provide the same optimal solution since no deviation is allowed and the level of variability no longer matters. If we fix budget and only focus on the ρ, it is obviously that smaller ρ results in higher optimal objective value. That is, the worst-case value for small ρ is actually better than the one for large ρ since the maximum deviation is smaller. The trajectories in Figure 4.4 appear piecewise linear because only integer budgets are calculated.

Details about relationships between probability of constraint violation and budget are included in the Figure 4.5. In Figure 4.5a, we show the probability of violation ϵ with respect to Γ_D^s. As budget increases, there are more protections toward constraint and the probability of violation decreases. When budget reaches the maximum value $|N|*|T|$, the constraint is completely protected and the probability of violation reduces to 0. Reversely, we can measure the minimum Γ_D^s with

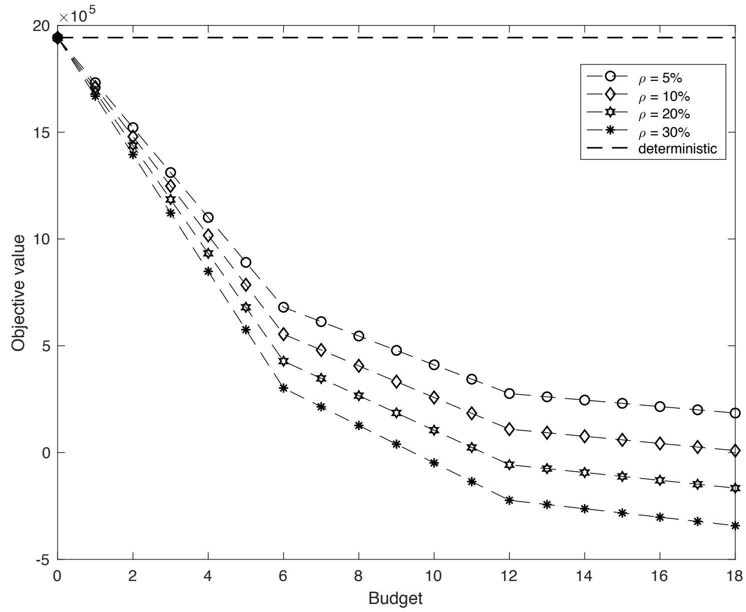
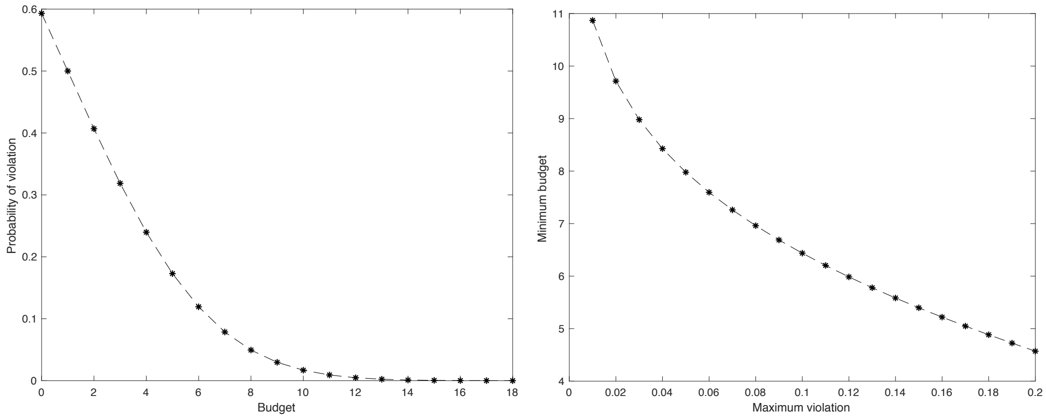


Figure 4.4: Optimal objective value with respect to different Γ_D^s and ρ

respect to ϵ by taking the inverse function. It is shown in Figure 4.5b, If the maximum violation probability is set to 0.17, then the minimum budget in order to maintain that violation probability is around 5.

In the Figure 4.6, we investigate how capacity of regular time production batch size (q) and ratio of regular vs overtime production (α) affect the optimal objective value. Clearly, bigger q value means more regular time production resource. In addition, overtime production quantity is bounded by the regular time production batch size and hence bigger q indicates more overtime production resource as well. It is shown in Figure 4.6 that if extra money can be invested, then production batch size q is certainly much more promising than overtime production α. When we fix q and only focus on the α, bigger α provides more overtime production resource and hence results in higher profit. When we fix α and vary q, it does not only provide more overtime production resource but also regular time production resource. It is why there is big improvement in the optimal profit when we vary q.

Next, we conduct the sensitivity analyses on parameters that associate with time resources such as total time availability cap_t, processing time pt_i and setup changeover time $st_{i,j}$. In Table 4.5, the sensitivity analysis for overall time capacity has been illustrated on the machine as well as utilization of time resources. U_i represents the time utilization of time period i. As we increases overall time availability, there is more regular time production resources and hence optimal profit increases. Note that there is no improvement when cap_t changes from 0.9 to 1 indicating we simply waste 10% of time resource. In addition, we observe that, as we reduce cap_t, U_5 and U_6 first approach 1 followed by U_2, U_3, then U_1 and U_4. It provides insights on how to invest time resources. Clearly, time period 5 and 6 have the highest priority since their utilizations approach to 1 first. Then time period 2, 3 followed by 1 and 4 should be invested if there is enough budget.



(a) ϵ as a function of Γ_D^s (b) Γ_D^s as a function of ϵ

Figure 4.5: Relationship between Γ_D^s and ϵ

Table 4.5: Optimal profit and time utilization (U_t) for different overall time capacities (cap_t)

Factor (cap_t)	Optimal profit	U_1	U_2	U_3	U_4	U_5	U_6
0.7	1906807	0.93	1	0.951	0.93	1	1
0.8	1931663	0.814	0.944	0.814	0.814	1	1
0.9	1941988	0.724	0.839	0.724	0.74	0.999	0.999
1	1941988	0.651	0.755	0.651	0.666	0.9	0.9

U_t: Utilization of overall time capacity in period t, $t = 1, \cdots, 6$

Processing time pt_i and setup changeover time $st_{i,j}$ also play important roles in the decision making process. Different pt_i and $st_{i,j}$ are tested in Figure 4.7. Since overall time availability is fixed, both pt_i and $st_{i,j}$ can affect regular time production plan. From Equation 4.8, we can see that increasing pt_i or $st_{i,j}$ reduces regular time production capacity. That is, when we increase factor, optimal profit decreases. Note that there is a point where optimal profit becomes negative meaning current production resources are not able to make profit due to large penalty cost. In addition, we can see that optimal profit is more sensitive to the processing time than setup cost. The main reason is that we assume setup takes place between products from different families. Intuitively, when we produce only one type of product, there is no setup changeover cost but processing cost always exists as long as there is production activity.

Finally, we study the sensitivity for parameters that associate with budget such as regular time processing cost, selling price, penalty and surplus cost. Selling price is originally 3 times more expensive than the regular production cost. That is, when factor is set to 5, pr_i becomes higher than the profit and optimal profit becomes negative. Smaller pr_i provides larger revenue and thus profits. Conversely, if we decrease c_i, profit becomes smaller. When factor is set to 0.3 meaning $c_i < pr_i$, optimal profit becomes negative. Since uncertain demand can vary in a predetermined interval where the nominal value and maximum deviation are specified. From Table 4.6, we can see

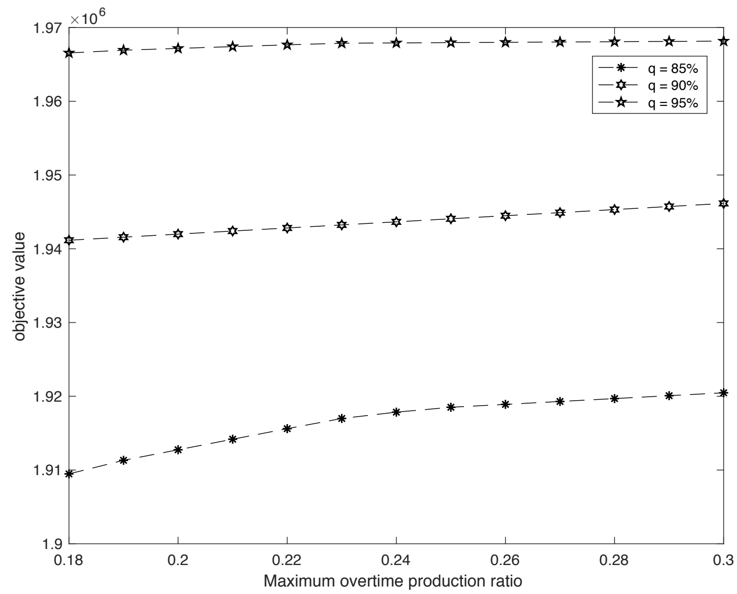


Figure 4.6: Optimal objective value with respect to different q and α

that production decisions becomes riskier since extra costs for unmet demand and excess amount of production increase as penalty and surplus costs increase. It should be noted that the matrix in Table 4.6 is symmetric with respect to the main diagonal since uncertain demand vary in a symmetric interval and hence the penalty and surplus costs have identical influence on the optimal profit.

Table 4.6: Optimal objective value as a function of penalty (pen) and surplus (sur) costs

		Surplus cost (sur)			
		0.5	1	1.5	2
Penalty cost (pen)	0.5	1496793	1354316	1283077	1240344
	1	1354316	1051597	877072	760721
	1.5	1283077	877072	606401	418141
	2	1240334	760721	418141	161205

4.4 Conclusion

In this paper, we study a multi-product, multi-period and capacitated lot-sizing and scheduling problem under demand and overtime production cost uncertainties. We adopt the HSRO approach of Keyvanshokooh et al. (2016) to maximize overall profit under demand and overtime processing cost uncertainties. We assume that overtime processing cost is predictable and hence historical

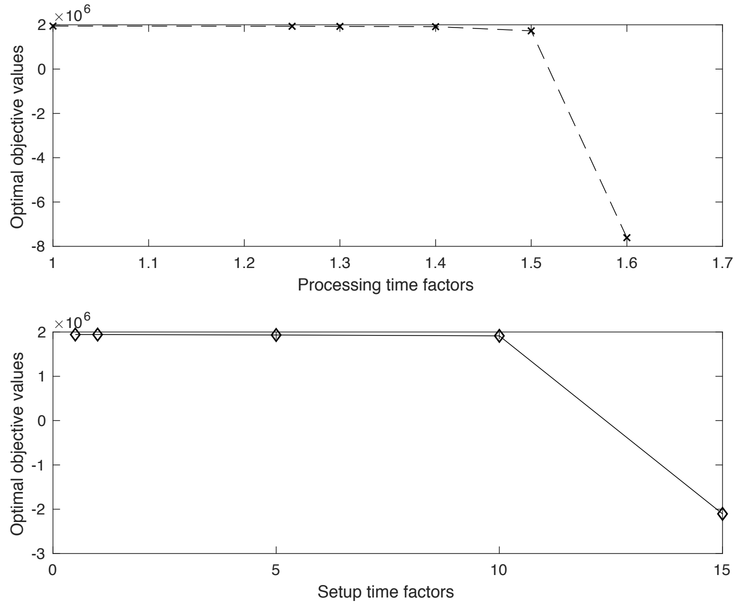


Figure 4.7: Optimal profit for different processing times (pt_i) and setup times ($st_{i,j}$)

data can be used to generate scenarios. Inside each overtime processing cost, polyhedral sets are introduced for uncertain demands due to their unpredictable characteristic. In the case study, braking production from an automotive company is used to illustrate and validate the proposed model and solution. Scenario stability tests have been conducted to identify proper scenario size. As scenario size increases, the objective value becomes more stable since scenarios have better representation of original distribution. When scenario sample size is 150, the relative difference between the maximum and minimum objective value is less than 0.3%. Then we carry out sensitivity analyses for parameters like budget Γ_D^s, constraint violation probability ϵ and time availability on the machine cap_t etc. Budget provides the level of robustness at the expense of profit. Higher Γ_D^s results in better protection against uncertainty, but the corresponding profit becomes lower. Constraint violation probability ϵ can be setup by decision makers as the maximum violation probability of constraints or calculated for given Γ_D^s. Concretely, ϵ can be written as a strictly decreasing function with respect to Γ_D^s. Time availability and utilization are also conducted to provide valuable insights. It can be shown that there are at least 10% waste of time resources. In addition, time periods 5 and 6 are bottleneck that can be improved upon if there is an increase in the production load. Impacts of other parameters like setup time, processing cost and selling price have also been studied in the paper.

In conclusion, we introduce a framework to investigate multiple source of uncertainties of varying characteristics in the scope of lot-sizing and scheduling production. Future research can be explored in the following directions. First, we assume that a particular setup can be taken place at most once in each time period while in reality, multiple setup times maybe allowed. It increases the complexity of a problem exponentially and hence heuristics can be considered. Second, a lot-sizing and scheduling problem is currently at least as difficult as solving one traveling salesman problem in each time period. Some valid inequalities can be applied for computational performance

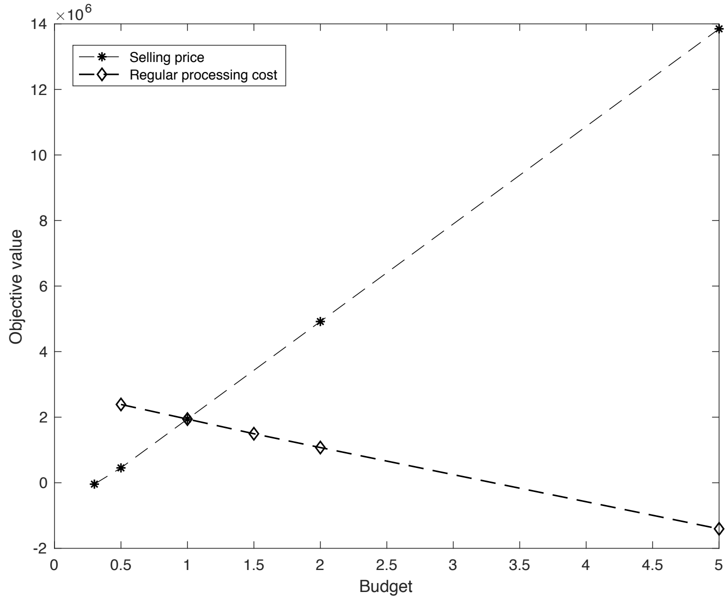


Figure 4.8: Optimal profit for different regular processing costs (pr_i) and selling prices (c_i)

comparisons when number of products increases. Last, demand is assumed to be independent from previous observations and this may not be true in other context where there are strong seasonalities and trends.

CHAPTER 5. A FUZZY MULTI-OBJECTIVE FACILITY LOCATION MODEL FOR CLOSED-LOOP SUPPLY CHAIN NETWORK DESIGN UNDER UNCERTAINTIES

The importance of considering forward and backward flows simultaneously in supply chain networks spurs an interest to develop closed-loop supply chain networks (CLSCN). Due to the expanded scope in the supply chain, designing CLSCN often faces significant uncertainties. This paper proposes a fuzzy multi-objective mixed integer linear programming model to deal with uncertain parameters in CLSCN. The two objective functions are minimization of overall system costs and minimization of negative environmental impact. Negative environmental impacts are measured and quantified through CO2 equivalent emission. Uncertainties include demand, return, scrap rate, manufacturing cost and negative environmental factors. The original formulation with uncertain parameters is firstly converted into a crisp model and then an aggregation function is applied to combine the objective functions. Numerical experiments have been carried out to demonstrate the effectiveness of proposed model formulation and solution approach. Sensitivity analyses on degree of feasibility, the weighing of objective functions and coefficient of compensation have been conducted.

5.1 Introduction

The increasing need for remanufacturing, the growing market competition, and the concern on negative environmental impacts have spurred significant interest in closed-loop supply chain network (CLSCN) adoption in manufacturing industry. In contrast to designing the forward and reverse material flows separately, the integrated system can achieve global optimality considering both flows in the supply chain. As pointed out by Klibi et al. (2010), the complex and dynamic nature of CLSCN creates a lot of uncertainties in the supply chain system and dramatically influences the overall performance of the logistics. The design of a supply chain network often involves long-term strategic decisions which have sustaining impact in business operations. Pishvaee et al. (2011) stated that opening/closing or upgrading a facility are capital intensive and time-consuming, and hence making any changes to those decisions in real time is often impossible. Therefore, it is essential to incorporate uncertainties into the design of CLSCN such that the decisions in the supply chain network configuration are efficient and robust.

Scenario-based stochastic programming, robust optimization, and fuzzy programming approaches have been widely applied in CLSCN to deal with uncertainties. Scenario-based stochastic programming is a powerful approach when probabilistic distribution information for the uncertain parameter is available. However, this approach has limitations (Bertsimas and Thiele, 2006; Gülpınar et al., 2013). First, in many real-world applications the decision maker may not have enough historical data, thus, estimating the accurate probabilistic distribution is impossible. For instance, estimating probabilistic distribution of demand of new product can be challenging. Second, an accurate approximation of probabilistic distribution may require a large amount of scenarios which increase the computational complexity. On the other hand, if scenario sample size is restricted for computational reasons, then the range of future realizations under which decisions are determined and evaluated is limited. To address these limitations, robust optimization has been introduced as an alternative approach to deal with uncertainty. Robust optimization handles uncertainties by solving robust counterpart over predetermined uncertainty sets. The robust counterpart is a worst-case formulation of the original problem in which worst-case is measured over all possible values that uncertain parameters may take in given convex sets. The main advantage of robust optimization in contrast to scenario-based stochastic programming is that only rough historical data is required to derive the uncertainty sets (Alem and Morabito, 2012). However, the main limitations are: First, only a few uncertain parameters were considered in robust optimization for CLSCN mainly due to reformulation as well as computational complexity(Hasani et al., 2012; Mirzapour Al-E-Hashem et al., 2011; Pishvaee et al., 2011). As stated by Prajogo and Olhager (2012), supply chain network design often involves decisions from multiple stakeholders and significant amount of uncertainties. Second, robust optimization assumes all uncertain coefficients belong to a predefined symmetric interval centered at the nominal value. This may not be true for some real-world applications in which uncertainties have highly skewed distributions. Third, robust optimization assumes uncertainty to affect only the constraint coefficients. It should be noted that a problem with uncertainties in the objective functions or right hand side of constraints requires reformulation and thus increase computational complexity. As an alternative, the main advantages of fuzzy programming are: First, this approach provides a framework to handle multiple uncertainties at the same time without increasing model complexity. Those uncertainties can affect not only left hand side of constraints but also right hand side of constraints as well as objective function. Second, this approach does not require complete information about uncertainty. Uncertainties in the fuzzy programming are dealt with triangular or trapezoidal membership function in which only rough data is required to determine the most pessimistic value, the most possible value and the most optimistic value. However, if the exact information of uncertainty is available, then the scenario-based stochastic programming is preferred. Third, this method provides degree of flexibility in constraints and degree of satisfaction level in the objective function, simultaneously (Dubois et al., 2003).

The idea of CLSCN was first proposed by Fleischmann et al. (2001) who found out the model with integration of forward and reverse flows could provide significant cost saving in contrast to separated decision making models. They presented a deterministic CLSCN model and discussed differences with traditional logistics setting. Similar deterministic CLSCN studies can be found in the following literatures(Ko and Evans, 2007; Lu and Bostel, 2007; Üster et al., 2007). Of the few recent studies that consider uncertainties in the CLSCN problem, most of them estimate the probabilistic distributions for the uncertain parameters and then apply scenario-based stochastic programming which samples scenarios from the probabilistic distributions followed by scenario reduction techniques (Lee and Dong, 2008; Listeş, 2007; Salema et al., 2007). Scenario-based stochastic programming is a powerful tool if there is enough historical data to fit an accurate probabilistic distribution for the random variables. However, data may not be sufficient to identify the underlying distribution in some real world applications. Robust optimization is an alternative approach to cope with uncertainties with rough historical data and related studies can be found in the following literatures (Hasani et al., 2012; Pishvaee et al., 2011). On top of that, there are papers that apply multiple methods, simultaneously. Keyvanshokooh et al. (2016) proposed a hybrid robust and stochastic programming approach for CLSCN in which demand, return and cost uncertainties have been studied. Vahdani et al. (2012) presented a bi-objective mathematical model for CLSCN with cost uncertainty. In order to solve the problem, they introduced a novel solution methodology by combining queuing theory, robust optimization and fuzzy multi-objective programming. To the best of authors' knowledge, these papers studied at most three types of uncertainties, namely demand, return and cost. However, CLSCN is a long-term planning problem and hence there are a large number of uncertainties in the decision making process. Fuzzy programming is an appropriate approach for problems with multiple uncertainties since: (1) the complexity of problem is independent of the number of uncertainties and (2) this method does not increase the number of objective functions or constraints. Pishvaee and Torabi (2010) introduced a fuzzy mathematical programming model for CLSCN with two objective functions: minimization of total costs and minimization of total delivery tardiness. Zarandi et al. (2011) considered uncertainties in the decision maker's aspiration levels as the objectives are imprecise. There are four different objective functions in the paper: first two objective functions aim to minimize the overall costs and the last two objective functions focus on the maximization of total service level. Jindal and Sangwan (2014) introduced a fuzzy mixed integer linear programming model for CLSCN with a single objective function which maximizes the overall profit. Kumar and Kumar (2013) compared a traditional supply chain network system with a closed-loop supply chain network system and made the following claim: The traditional supply chain seeks to reduce the cost and improve the efficiency while CLSCN aims to lower the consumption of resources and decrease the emissions of pollutants so as to maximize the economic benefits. Amin and Zhang (2013) emphasized the importance of considering environmental impact in CLSCN because environmental protection is included in the both internal and external management.

In this paper, we propose a mathematical model for a single-product, multi-period and capacitated CLSCN. The tactical decisions include determining flows among the facilities while strategic decisions involve facility location selection. The major contributions can be summarized as follows:

- A novel multi-objective CLSCN model is proposed. The goal is to minimize the overall system costs and negative environmental impact which is measured and quantified by CO2 equivalent emission.
- Fuzzy programming is applied to convert original formulation with uncertain parameters into a crisp model. After that, an aggregation function is applied to integrate and evaluate the two different objective functions.

The remainder of this paper is organized as follows: The problem statement and model formulation are defined in section 5.2. The equivalent crisp model as well as solution approach are presented in section 5.3. The computational experiments and sensitivity analyses are included in section 5.4, and managerial insights are derived. Finally, section 5.5 concludes the paper with major findings and points out future research directions.

5.2 Problem definition and formulation

5.2.1 Problem statement

As shown in Figure 5.1, the network design problem studied in this paper is a single product, multi-period and capacitated CLSCN including manufacturing plants, distribution, collection, recovery and disposal centers. Given the customer demands, the goal is to find the optimal facility locations as well as materials flows such that overall system costs and negative environmental impacts are minimized. The negative environmental impacts are measured and quantified by CO2 equivalent emission. We assume that facilities with higher capital investments have environmental friendly machines and clean technology, therefore, negative environmental impacts are inversely proportional to capital investment. This supply chain system consists of both forward and backward flows. In the forward network, manufacturing plants produce and transport products to distribution centers and then to customers. In the backward network, defective/used products are collected from customers and shipped to collection centers. After a quality examination process, returned products are classified into two different categories depending on their conditions. The recoverable and scrapped items are sent to recovery and disposal centers, respectively. After appropriate processing, recovered items are sent back to distribution centers and reenter the forward network.

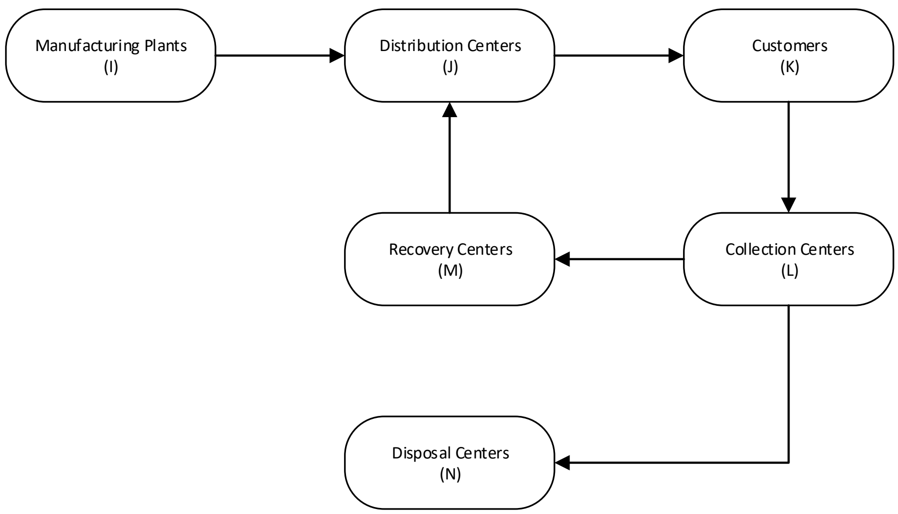


Figure 5.1: Closed-loop supply chain configuration

In this paper, we assume that products are fairly new to the market and therefore not enough historical data are available to estimate the distributions of demand, return, and processing cost etc. On the other hand, network infrastructure information such as fixed cost, maximum capacity and transportation cost are assumed to be known. Because there are multiple uncertain parameters with limited amount of historical data, we decide to use fuzzy programming for its modeling and computational efficiency (Liang, 2006).

5.2.2 Model formulation

The following mathematical notations have been used in the formulation of the CLSCN. Parameters with uncertainty are represented with a tilde sign on. Demand volume, return volume, average scrap rate and unit processing cost at different facilities are considered to be uncertain. In addition, we consider uncertainties in negative environmental impact through CO2 equivalent emission. Parameters like facility fixed cost, facility maximum capacity, and transportation cost are considered to be known and fixed.

Sets:

i set of potential locations for manufacturing plants $i = 1 \cdots I$

j set of potential locations for distribution centers $j = 1 \cdots J$
k set of fixed locations of customers $k = 1 \cdots K$
l set of potential locations for collection centers $l = 1 \cdots L$
m set of potential locations for recovery centers $m = 1 \cdots M$
n set of potential locations for disposal centers $n = 1 \cdots N$
t set of time periods $t = 1 \cdots T$

Parameters:

$\tilde{d}_{kt}$: demand volume of customer k in time period t

$\tilde{\omega}_{kt}$: percentage of return (probability of items being returned) from customer k in time period t

$\tilde{\eta}_t$: mean scrap rate in time period t

α_i: fixed cost of building manufacturing plant i

f_j: fixed cost of building distribution center j

g_l: fixed cost of building collection center l

a_n: fixed cost of building disposal center n

b_m: fixed cost of building recovery center m

co_{ij}: unit product shipping cost from manufacturing plant i to distribution center j

cu_{jk}: unit product shipping cost from distribution center j to customer k

cq_{kl}: unit product shipping cost from customer k to collection center l

cp_{lm}: unit product shipping cost from collection center l to recovery center m

cs_{ln}: unit product shipping cost from collection center l to disposal center n

ch_{mj}: unit product shipping cost from recovery center m to distribution center j

$\tilde{\rho}_i$: unit production cost at manufacturing plant i

$\tilde{\varphi}_j$: unit processing cost at distribution center j

$\tilde{\beta}_l$: unit processing cost at collection center l

$\tilde{\tau}_m$: unit reproduction cost at recovery center m

pr_i: maximum capacity of manufacturing plant i in each time period

px_j: maximum capacity of distribution center j in each time period

py_l: maximum capacity of collection center l in each time period

pz_m: maximum capacity of recovery center m in each time period

pw_n: maximum capacity of disposal center n in each time period

$e\tilde{r}_i$: negative environmental impact factor for opening a manufacturing plant at location i

$e\tilde{x}_j$: negative environmental impact factor for opening a distribution center at location j

$e\tilde{y}_l$: negative environmental impact factor for opening a collection center at location l

$e\tilde{z}_m$: negative environmental impact factor for opening a recovery center at location m

$e\tilde{w}_n$: negative environmental impact factor for opening a disposal center at location n

Decision variables:

O_{ijt}: volume of products transported from manufacturing plant i to distribution center j in time period t

U_{jkt}: volume of products transported from distribution center j to customer k in time period t

Q_{klt}: volume of returned items transported from customer k to collection center l in time period t

P_{lmt}: volume of recoverable items transported from collection center l to recovery center m in time period t

S_{lnt}: volume of scrapped items transported from collection center l to disposal center n in time period t

H_{mjt}: volume of recovered items transported from recovery center m to distribution center j in time period t

R_i: 1 if a manufacturing plant is built at location i and 0 otherwise

X_j: 1 if a distribution center is built at location j and 0 otherwise

Y_l: 1 if a collection center is built at location l and 0 otherwise

Z_m: 1 if a recovery center is built at location m and 0 otherwise

W_n: 1 if a disposal center is built at location n and 0 otherwise

With these defined notations, the CLSCN problem can be constructed as follows:

5.2.2.1 Objective functions

$$\begin{aligned}\min \quad \zeta_1 = & \sum_i \alpha_i \cdot R_i + \sum_j f_j \cdot X_j + \sum_l g_l \cdot Y_l + \sum_n a_n \cdot W_n + \sum_m b_m \cdot Z_m \\ & + \sum_{i,j,t} (\tilde{\rho}_i + co_{ij}) \cdot O_{ijt} + \sum_{j,k,t} (\tilde{\varphi}_j + cu_{jk}) \cdot U_{jkt} + \sum_{k,l,t} cq_{kl} \cdot Q_{klt} \\ & + \sum_{l,m,t} (\tilde{\beta}_l + cp_{lm}) \cdot P_{lmt} + \sum_{l,n,t} (\tilde{\beta}_l + cs_{ln}) \cdot S_{lnt} \\ & + \sum_{m,j,t} (\tilde{\tau}_m + ch_{mj}) \cdot H_{mjt}\end{aligned} \tag{5.1}$$

$$\min \quad \zeta_2 = \sum_i \tilde{er}_i \cdot R_i + \sum_j \tilde{ex}_j \cdot X_j + \sum_l \tilde{ey}_l \cdot Y_l + \sum_m \tilde{ez}_m \cdot Z_m + \sum_n \tilde{ew}_n \cdot W_n \tag{5.2}$$

The strategic decisions in this CLSCN design include the numbers as well as locations of manufacturing plants, distribution, collection, recovery and disposal centers. In addition, decisions need to be made on the flow volume between facilities in each time period. Two objective functions are: minimization of overall system costs and minimization of negative environmental impact. Overall system costs include fixed costs, transportation costs and manufacturing costs. We use CO2 equivalent emission to measure and quantify negative environmental impact. The inverse relationship between capital investment costs (α_i, f_j, g_l, a_n, b_m) and CO2 equivalent emission is embedded in the negative environmental impact factors ($\tilde{er}_i$, $\tilde{ex}_j$, $\tilde{ey}_l$, $\tilde{ez}_m$, $\tilde{ew}_n$). Notably, Equation 5.1 and 5.2 are in conflict with each other. That is, higher value in one objective function results in lower value in another one and hence optimizing the CLSCN requires a trade-off between these two contradictory objective functions.

5.2.2.2 Constraints

$$\sum_j U_{jkt} \geq \tilde{d}_{kt} \qquad \forall k, t \tag{5.3}$$

$$\sum_l Q_{klt} \geq \tilde{\omega}_{kt} \cdot \tilde{d}_{kt-1} \qquad \forall k, t \tag{5.4}$$

$$\sum_i O_{ijt} + \sum_m H_{mjt} = \sum_k U_{jkt} \qquad \forall j, t \tag{5.5}$$

$$\tilde{\eta}_t \cdot \sum_k Q_{klt} = \sum_n S_{lnt} \qquad \forall l, t \tag{5.6}$$

$$(1 - \tilde{\eta}_t) \cdot \sum_k Q_{klt} = \sum_m P_{lmt} \qquad \forall l, t \tag{5.7}$$

$$\sum_j H_{mjt} = \sum_l P_{lmt} \qquad \forall m, t \tag{5.8}$$

$$\sum_j O_{ijt} \leq R_i \cdot pr_i \qquad \forall i, t \tag{5.9}$$

$$\sum_i O_{ijt} + \sum_m H_{mjt} \leq X_j \cdot px_j \qquad \forall j, t \tag{5.10}$$

$$\sum_k Q_{klt} \leq Y_l \cdot py_l \qquad \forall l, t \tag{5.11}$$

$$\sum_l P_{lmt} \leq Z_m \cdot pz_m \qquad \forall m, t \tag{5.12}$$

$$\sum_l S_{lnt} \leq W_n \cdot pw_n \qquad \forall n, t \tag{5.13}$$

$$R_i, X_j, Y_l, Z_m, W_n \in \{0, 1\} \qquad \forall i, j, l, m, n \tag{5.14}$$

$$O_{ijt}, U_{jkt}, Q_{klt}, P_{lmt}, S_{lnt}, H_{mjt} \geq 0 \qquad \forall i, j, k, l, m, n, t \tag{5.15}$$

We assume that all demands must be satisfied and no backlog is allowed. Equation 5.3 ensures the demands are satisfied. Equation 5.4 makes sure return products are collected and shipped to the collection centers. Equation 5.5 - 5.8 are flow balance equations which assure flow balance at distribution, collection and recovery centers. Equation 5.9 - 5.13 are maximum capacity constraints which enforce, in each time period, the difference between incoming and outgoing flows for each facility is no larger than the maximum capacity. Equation 5.14 indicates all facility location variables have to be binary and Equation 5.15 indicates all flow variables have to be non-negative.

5.3 The proposed solution method

The proposed CLSCN design formulation is a mixed integer linear programming problem with multi-objective functions. Since membership functions are used to capture the uncertainties, we transform the original model into an equivalent crisp model in the first stage. In the second stage, we combine two objective functions and solve the crisp model to obtain solutions.

5.3.1 The equivalent auxiliary crisp model

Multiple approaches have been proposed in the literature to handle formulation with uncertain parameters in both constraints and objective functions (Inuiguchi and Ramık, 2000; Jiménez et al., 2007; Wang and Liang, 2005). In this paper, we adopt Jiménez et al. (2007) approach. The major advantage of their method is that it does not introduce extra objective functions or constraints and the whole problem remains linear. This approach is based on the concept of expected interval and expected value of a fuzzy parameters.

Assume $\tilde{c}$ is a triangular fuzzy number whose membership function $\mu_{\tilde{c}}$ can be represented by the following equation:

$$\mu_{\tilde{c}}(x) = \begin{cases} f_c(x) = \frac{x-c^p}{c^m-c^p} & \text{if } c^p \leq x \leq c^m \\ 1 & \text{if } x = c^m \\ g_c(x) = \frac{c^o-x}{c^o-c^m} & \text{if } c^m \leq x \leq c^o \\ 0 & \text{if } x \leq c^p \text{ or } x \geq c^o \end{cases} \tag{5.16}$$

where c^p, c^m and c^o indicate the most pessimistic value, the most possible value and the most optimistic value. These membership functions can be stated as the degree of occurrence of parameters which are usually determined based on historical data and experts' knowledge. According to Jiménez et al. (2007), the expected value (EV) and expected interval (EI) of a triangular fuzzy number $\tilde{c}$ can be defined as follow:

$$EV(\tilde{c}) = \frac{E_1^c + E_2^c}{2} = \frac{c^p + 2c^m + c^o}{4} \tag{5.17}$$

$$EI(\tilde{c}) = [E_1^c, E_2^c] = [\int_0^1 f_c^{-1}(x)dx, \int_0^1 g_c^{-1}(x)dx] = [\frac{1}{2}(c^p + c^m), \frac{1}{2}(c^m + c^o)] \tag{5.18}$$

Two problems need to be addressed when the formulation contain uncertain parameters: (1) How to define a feasible solution when the constraints have fuzzy parameters; (2) How to define an optimal solution when the objective functions have fuzzy coefficients. Multiple approaches for ranking fuzzy numbers can be found in the following literatures (Rommelfanger and Słowiński, 1998; Sakawa, 2013). Different properties have been studied to justify ranking approaches such as robustness and distinguishability.

According to Jiménez et al. (2007), any pair of fuzzy number $\tilde{a}$ and $\tilde{b}$, the degree in which $\tilde{a}$ is larger than $\tilde{b}$ can be stated as follows:

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^b < 0 \\ \frac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in [E_1^a - E_2^b, E_2^a - E_1^b] \\ 1 & \text{if } E_1^a - E_2^b > 0 \end{cases} \tag{5.19}$$

where $[E_1^a, E_2^a]$ and $[E_1^b, E_2^b]$ are the expected interval of fuzzy parameters $\tilde{a}$ and $\tilde{b}$. Expression $\mu_M(\tilde{a}, \tilde{b}) \geq \alpha$ or $\tilde{a} \geq_\alpha \tilde{b}$ can be viewed as fuzzy parameter $\tilde{a}$ is no smaller than $\tilde{b}$ in degree α. Similar ranking approaches can be found in the following literatures (Fortemps and Roubens, 1996;

González, 1990). According to Parra et al. (2005), for any pair of fuzzy parameters $\tilde{a}$ and $\tilde{b}$, we say that these two fuzzy parameters are equivalent in degree of α if Equation 5.20 holds.

$$\tilde{a} \geq_{\frac{\alpha}{2}} \tilde{b} \quad \text{and} \quad \tilde{a} \leq_{\frac{\alpha}{2}} \tilde{b} \tag{5.20}$$

$\tilde{a} \leq_{\frac{\alpha}{2}} \tilde{b}$ indicates that $\tilde{b}$ is larger than or equal to $\tilde{a}$ at least in degree $\frac{\alpha}{2}$. Equivalently, it also indicates that $\tilde{a}$ is larger than or equal to $\tilde{b}$ at most in degree $1 - \frac{\alpha}{2}$. Therefore, Equation 5.20 can be reformulated as follow:

$$\frac{\alpha}{2} \leq \mu_M(\tilde{a}, \tilde{b}) \leq 1 - \frac{\alpha}{2} \tag{5.21}$$

Let's consider a fuzzy mathematical programming Equation 5.22 in which all coefficients and parameters are defined as triangular fuzzy numbers. It should be noted that deterministic objective functions and constraints remain unchanged.

$$\begin{array}{llcll} \min\limits_{x} & \tilde{c}^T x & & & \\ \text{s.t.} & \tilde{a}_i x & \geq & \tilde{b}_i & i = 1, \cdots, l \\ & \tilde{a}_i x & = & \tilde{b}_i & i = l+1, \cdots, m \end{array} \tag{5.22}$$

According to Zimmermann (1978) approach, a fuzzy solution is given by the intersection of all fuzzy objective functions and constraints. A solution x is feasible in degree α if $\min_{i=1,\cdots,m}[\mu_M(\tilde{a}_i x, \tilde{b}_i)] = \alpha$. Using Equation 5.19 and 5.21, fuzzy constraints $\tilde{a}_i x \geq \tilde{b}_i$ and $\tilde{a}_i x = \tilde{b}_i$ can be rewritten as follows:

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \geq \alpha \quad i = 1, \cdots, l \tag{5.23}$$

$$\frac{\alpha}{2} \leq \frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \leq 1 - \frac{\alpha}{2} \quad i = l+1, \cdots, m \tag{5.24}$$

Equation 5.23 and 5.24 can be reformulated as follows:

$$[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}]x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i} \quad i = 1, \cdots, l \tag{5.25}$$

$$[(1-\frac{\alpha}{2})E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}]x \geq \frac{\alpha}{2}E_2^{b_i} + (1-\frac{\alpha}{2})E_1^{b_i} \quad i = l+1, \cdots, m \tag{5.26}$$

$$[\frac{\alpha}{2}E_2^{a_i} + (1-\frac{\alpha}{2})E_1^{a_i}]x \leq (1-\frac{\alpha}{2})E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i} \quad i = l+1, \cdots, m \tag{5.27}$$

Similarly, a feasible solution x^o is α - acceptable optimal solution if and only if for all feasible solution x, the following equation holds:

$$\tilde{c}^t x \geq_{\frac{1}{2}} \tilde{c}^t x^o \tag{5.28}$$

That is, x^o is a better solution in terms of objective value at least in degree $\frac{1}{2}$ as opposed to other feasible solution x. Equation 5.28 can be expressed as $\mu_M(\tilde{c}^t x, \tilde{c}^t x^o) \geq \frac{1}{2}$. After plugging in to Equation 5.23, we get the following equation:

$$\frac{E_1^{c^t x} + E_2^{c^t x}}{2} \geq \frac{E_1^{c^t x^o} + E_2^{c^t x^o}}{2} \tag{5.29}$$

The equivalent crisp α - acceptable model of Equation 5.22 can be reformulated as follows:

$$\begin{array}{llcll} \min\limits_{x} & EV(\tilde{c})x & & & \\ \text{s.t.} & [(1-\alpha)E_2^{a_i}+\alpha E_1^{a_i}]x & \geq & \alpha E_2^{b_i}+(1-\alpha)E_1^{b_i} & i=1,\cdots,l \\ & [(1-\frac{\alpha}{2})E_2^{a_i}+\frac{\alpha}{2}E_1^{a_i}]x & \geq & \frac{\alpha}{2}E_2^{b_i}+(1-\frac{\alpha}{2})E_1^{b_i} & i=l+1,\cdots,m \\ & [\frac{\alpha}{2}E_2^{a_i}+(1-\frac{\alpha}{2})E_1^{a_i}]x & \leq & (1-\frac{\alpha}{2})E_2^{b_i}+\frac{\alpha}{2}E_1^{b_i} & i=l+1,\cdots,m \end{array} \quad (5.30)$$

Using Equation 5.30, the equivalent crisp CLSCN problem can be rewritten as follows:

$$\begin{aligned} \min \quad \zeta_1 = & \sum_i \alpha_i \cdot R_i + \sum_j f_j \cdot X_j + \sum_l g_l \cdot Y_l + \sum_n a_n \cdot W_n + \sum_m b_m \cdot Z_m \\ & + \sum_{i,j,t} (\frac{\rho_i^p + 2\rho_i^m + \rho_i^o}{4} + co_{ij}) \cdot O_{ijt} + \sum_{j,k,t} (\frac{\varphi_j^p + 2\varphi_j^m + \varphi_j^o}{4} + cu_{jk}) \cdot U_{jkt} \\ & + \sum_{l,m,t} (\frac{\beta_l^p + 2\beta_l^m + \beta_l^o}{4} + cp_{lm}) \cdot P_{lmt} + \sum_{l,n,t} (\frac{\beta_l^p + 2\beta_l^m + \beta_l^o}{4} + cs_{ln}) \cdot S_{lnt} \\ & + \sum_{m,j,t} (\frac{\tau_m^p + 2\tau_m^m + \tau_m^o}{4} + ch_{mj}) \cdot H_{mjt} + \sum_{k,l,t} cq_{kl} \cdot Q_{klt} \end{aligned} \quad (5.31)$$

$$\begin{aligned} \min \quad \zeta_2 = & \sum_i \frac{er_i^p + 2er_i^m + er_i^o}{4} \cdot R_i + \sum_j \frac{ex_j^p + 2ex_j^m + ex_j^o}{4} \cdot X_j \\ & + \sum_l \frac{ey_l^p + 2ey_l^m + ey_l^o}{4} \cdot Y_l + \sum_m \frac{ez_m^p + 2ez_m^m + ez_m^o}{4} \cdot Z_m \\ & + \sum_n \frac{ew_n^p + 2ew_n^m + ew_n^o}{4} \cdot W_n \end{aligned} \quad (5.32)$$

$$\sum_j U_{jkt} \geq \alpha \cdot (\frac{d_{kt}^m + d_{kt}^o}{2}) + (1-\alpha) \cdot (\frac{d_{kt}^p + d_{kt}^m}{2}) \qquad \forall k,t \quad (5.33)$$

$$\begin{aligned} & \sum_l Q_{klt} \geq \alpha \cdot (\frac{\omega_{kt}^m \cdot d_{kt-1}^m + \omega_{kt}^o \cdot d_{kt-1}^o}{2}) \\ & + (1-\alpha) \cdot (\frac{\omega_{kt}^p \cdot d_{kt-1}^p + \omega_{kt}^m \cdot d_{kt-1}^m}{2}) \qquad \forall k,t \end{aligned} \quad (5.34)$$

$$(\frac{\alpha}{2} \cdot \frac{\eta_t^m + \eta_t^o}{2} + (1-\frac{\alpha}{2}) \cdot \frac{\eta_t^p + \eta_t^m}{2}) \cdot \sum_k Q_{klt} \leq \sum_n S_{lnt} \qquad \forall l,t \quad (5.35)$$

$$((1-\frac{\alpha}{2}) \cdot \frac{\eta_t^m + \eta_t^o}{2} + \frac{\alpha}{2} \cdot \frac{\eta_t^p + \eta_t^m}{2}) \cdot \sum_k Q_{klt} \geq \sum_n S_{lnt} \qquad \forall l,t \quad (5.36)$$

$$(1-\frac{\alpha}{2} \cdot \frac{\eta_t^m + \eta_t^p}{2} - (1-\frac{\alpha}{2}) \cdot \frac{\eta_t^o + \eta_t^m}{2}) \cdot \sum_k Q_{klt} \leq \sum_m P_{lmt} \qquad \forall l,t \quad (5.37)$$

$$(1-(1-\frac{\alpha}{2})\cdot\frac{\eta_t^m+\eta_t^p}{2}-\frac{\alpha}{2}\cdot\frac{\eta_t^o+\eta_t^m}{2})\cdot\sum_k Q_{klt}\geq\sum_m P_{lmt}\qquad\forall l,t \tag{5.38}$$

It should be noted that Equation 5.5, 5.8 - 5.15 in the original formulation do not contain fuzzy parameters and hence remain unchanged in this formulation.

5.3.2 The fuzzy solution approach

Fuzzy mathematical programming has been widely used to solve multi-objective problems due to its' capability in quantifying the satisfaction level of each objective function. The very first fuzzy multi-objective solution approach was proposed by Zimmermann (1978), called max - min approach. The basic idea of this approach is to introduce an auxiliary variable λ, and then maximize λ given λ smaller than or equal to all objective values. However, this approach is not efficient and solution may not be unique (Lai and Hwang, 1993; Li et al., 2006a). In addition, this approach does not consider the relative importance of each objective function. Tiwari et al. (1987) proposed an additive model in which the relative importance of each objective function is considered, but the ratio of satisfaction level does not necessary match up with the relative importance level for the decision makers. In this paper, we adopt the approaches that proposed by Torabi and Hassini (2008).

In order to introduce this multi-objective aggregation function, we first define a linear membership function for each objective. This function can be viewed as the satisfaction level of each objective function. The linear membership function for a minimization objective can be defined as follows:

$$\mu_{\zeta_1}(x)=\begin{cases}1 & \text{for } \zeta_1(x)\leq\zeta_1^-\\ \frac{\zeta_1^+-\zeta_1(x)}{\zeta_1^+-\zeta_1^-} & \text{for } \zeta_1^-\leq\zeta_1(x)\leq\zeta_1^+\\ 0 & \text{for } \zeta_1(x)\geq\zeta_1^+\end{cases} \tag{5.39}$$

Similarly, the linear membership function for a maximization objective can be defined as follows:

$$\mu_{\zeta_2}(x)=\begin{cases}1 & \text{for } \zeta_2(x)\geq\zeta_2^+\\ \frac{\zeta_2(x)-\zeta_2^-}{\zeta_2^+-\zeta_2^-} & \text{for } \zeta_2^-\leq\zeta_2(x)\leq\zeta_2^+\\ 0 & \text{for } \zeta_2(x)\leq\zeta_2^-\end{cases} \tag{5.40}$$

It should be noted that the linear membership Equation 5.39 is used since both ζ_1 and ζ_2 are minimization functions. In this paper, there are two objective functions in the decision making problem. However, this approach can be easily generalized to problems with more than two objective functions. Given an α value, ζ_1^- and ζ_2^+ are obtained by solving the multi-objective problem as a single objective problem using only one objective function. Assuming the optimal solutions are x_1^* and x_2^*, respectively. Then, ζ_1^+ and ζ_2^- can be obtained by using the following expressions: $\zeta_1^+=\zeta_1(x_2^*)$ and $\zeta_2^-=\zeta_2(x_1^*)$.

The aggregation function can be expressed as follows (Torabi and Hassini, 2008):

$$\begin{array}{lll} \max_{x,\lambda_o} \lambda(x) & = \gamma\lambda_o + (1-\gamma)\sum_h \theta_h \mu_h(x) & \\ \text{s.t.} & \lambda_o \leq \mu_h(x) \qquad h = 1,2 & \\ & x \in F(x) \quad \text{and} \quad \gamma \in [0,1] & \end{array} \tag{5.41}$$

where $\mu_1(x)$ and $\mu_2(x)$ are the linear membership functions of two objective functions and F(x) denotes the feasible region of equivalent crisp model. In the result, $\lambda_o = \min_h\{\mu_h(x)\}$ indicates minimum satisfaction level of all objective functions. γ and θ_h indicate the coefficient of compensation and the relative importance of hth objective function.

5.4 Computational experiments

To demonstrate and validate the proposed model and solution technique, numerical experiments have been implemented and the results are shown in this section. The numerical example includes two potential locations for manufacturing plants, four potential locations for distribution centers, five fixed locations of customers, three potential locations for collection centers, two potential locations for disposal centers, and twelve time periods. The details can be found in the following literatures (Fahimnia et al., 2013; Krikke et al., 2003; Pishvaee and Torabi, 2010). To generate the triangular fuzzy parameters, three prominent points (the most likely value, the most pessimistic value and the most optimistic value) need to be estimated for each uncertain parameter. The most likely value (c^m) is first generated randomly using the uniform distribution. Subsequently, the corresponding most pessimistic value (c^p) and the most optimistic value (c^o) are determined, without loss of generality, by multiplying 0.8 and 1.2, respectively (Selim and Ozkarahan, 2008).

Besides these uncertainties, we also consider negative environmental impact uncertainty through CO2 equivalent emission. The most likely values for $e\tilde{r}_i$, $e\tilde{x}_j$, $e\tilde{y}_l$, $e\tilde{z}_m$ and $e\tilde{w}_n$ are set inversely proportional to the capital investment. This is based on the assumption that the environmental friendly facilities have higher capital investment due to additional expense on environmental friendly machines and clean technologies. Under this assumption, the two objective functions Equation 5.31 and 5.32 become conflict with each other since the first objective function tends to minimize overall system costs by opening economic facilities and second objective function aims to minimize negative environmental impact by opening more expensive facilities. Equation 5.41 is then applied to not only balance two contradictory objective functions but also provide a lower bound on the minimum satisfaction in the objective functions.

5.4.1 Sensitivity analysis on α

In order to determine Equation 5.41, the linear membership functions should be applied for Equation 5.31 and 5.32 by testing the range of their objective values. Table 5.1 shows the sensitivity analysis on α. $W_1^{\alpha-PIS}$ is the optimal objective value (minimum overall system costs) for Equation 5.31 at each level of feasibility α. Similarly, $W_2^{\alpha-PIS}$ is the optimal objective values (minimum negative environmental impact) for Equation 5.32 at each level of feasibility α. Meanwhile, we obtain the optimal decisions $x_1^{\alpha-PIS}$ and $x_2^{\alpha-PIS}$, respectively. $W_1^{\alpha-NIS}$ and $W_2^{\alpha-NIS}$ are derived by plugging the optimal decision $x_2^{\alpha-PIS}$ into Equation 5.31 and optimal decision $x_1^{\alpha-PIS}$ into Equation 5.32. For example when $\alpha = 0.5$, the minimum overall system cost is \$1,635,098 and the maximum overall system cost is \$1,695,098. Corresponding annual minimum negative environmental impact is 1700 tons and annual maximum negative environmental impact is 2000 tons. As shown in Figure 5.2, if the values are smaller than the optimal objective values (\$1,635,098 and 1700 tons), then the decision maker is 100% satisfied with the solution. If the values are greater then the worst objective values (\$1,695,098 and 2000 tons), then the decision maker is 0% satisfied with the solution. Between the best and the worst objective values, the level of satisfaction decreases as objective value increases since both ζ_1 and ζ_2 are minimization functions. The goal is to find a balance point between two conflicting objective functions based on the decision maker's preference. It should be noted that greater α results in more robust solution and hence objective values ($W_1^{\alpha-PIS}$, $W_1^{\alpha-NIS}$, $W_2^{\alpha-PIS}$, $W_2^{\alpha-NIS}$) increase as α increases. When α increases from 0.6 to 0.7, there are tremendous increments in both overall system costs and negative environmental impact due to network configuration upgrades. It points out the fact that the decision makers need to not only balance the objective functions but also focus on the quality of the solutions.

Table 5.1: Sensitivity analysis on degree of feasibility (α)

α	$W_1^{\alpha-PIS}$ (\$)	$W_1^{\alpha-NIS}$ (\$)	$W_2^{\alpha-PIS}$ (tons)	$W_2^{\alpha-NIS}$ (tons)
0.1	1,400,372	1,690,324	1700	2400
0.2	1,451,849	1,691,516	1700	2300
0.3	1,483,329	1,692,709	1700	2200
0.4	1,534,811	1,693,903	1700	2100
0.5	1,635,098	1,695,098	1700	2000
0.6	1,656,294	1,696,362	1700	1900
0.7	1,907,492	2,047,492	2200	2600
0.8	1,908,691	2,048,691	2200	2600
0.9	2,069,891	2,259,891	2500	3100

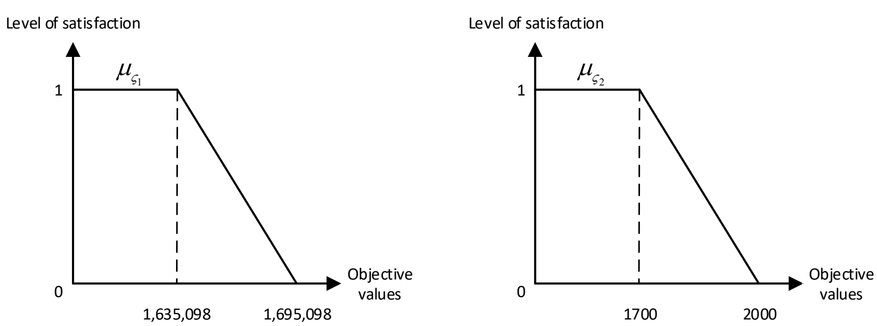


Figure 5.2: Linear membership functions for ζ_1 and ζ_2 when $\alpha = 0.5$

5.4.2 Sensitivity analysis on θ_1 and θ_2

The next step is to construct the linear membership functions $\mu_{\zeta_1}(x)$ and $\mu_{\zeta_2}(x)$ for a given α value. Let's set $\alpha = 0.5$, then the linear membership functions can be expressed as follows:

$$\mu_{\zeta_1}(x_1) = \frac{1,695,098 - x_1}{60,000} \qquad \text{and} \qquad \mu_{\zeta_2}(x_2) = \frac{2000 - x_2}{300}$$

Where x_1 and x_2 are the objective values for the crisp model. Notice that $x_1 \in [1635098, 1695098]$ and $x_2 \in [1700, 2000]$. The denominator of linear membership function is obtained by identifying the range of objective values (e.g. 1,695,098 - 1,635,098 = 60,000 and 2000 - 1700 = 300). Intuitively, the level of satisfaction decreases as x_1 and x_2 increase because they are minimization functions. The sensitivity analysis on the importance of objective functions is shown in Table 5.2. θ_1 is the importance of the first objective function ζ_1 (overall system costs) and θ_2 is the importance of the second objective function ζ_2 (negative environmental impact). Given the fact that $\theta_1 + \theta_2 = 1$, increasing θ_1 while decreasing θ_2 indicates the decision maker tends to put more focus on the overall system costs and less focus on the negative environmental impact. In Table 5.2, W_1 is the optimal objective value for ζ_1 and W_2 is the optimal objective value for ζ_2. μ_{W_1} and μ_{W_2} are the level of satisfaction for two objective functions, respectively. λ_0 is the minimum level of satisfaction: $\lambda_0 = \min(\mu_{W_1}, \mu_{W_2})$. When $\theta_1 = 0.9$ and $\theta_2 = 0.1$, corresponding $\mu_{W_1} = 1$ and $\mu_{W_2} = 0$. It indicates that the decision maker is 100% satisfied with the overall system costs and 0% satisfied with the negative environmental impact. This parameter setup makes the solution indifferent to traditional supply chain network system because it does not consider environmental impact and only care about the overall system costs. As θ_1 decreases, the decision makers put more and more focus on the environmental impact, therefore, W_1 increases and W_2 decreases. Notice that μ_{W_1}, μ_{W_2} and λ_0 are insensitive to parameter changes in θ_1 and θ_2. Apparently, minimum satisfaction λ_0 is fairly low across all θ_1 and θ_2 combinations and this motivates us to investigate how γ affect the model solution.

Table 5.2: Sensitivity analysis on the importance of objective functions (θ_1 and θ_2)

	W_1 (\$)	W_2 (tons)	μ_{W_1}	μ_{W_2}	λ_0
$\theta_1 = 0.1$, $\theta_2 = 0.9$	1,675,098	1800	1/3	2/3	1/3
$\theta_1 = 0.2$, $\theta_2 = 0.8$	1,675,098	1800	1/3	2/3	1/3
$\theta_1 = 0.3$, $\theta_2 = 0.7$	1,675,098	1800	1/3	2/3	1/3
$\theta_1 = 0.4$, $\theta_2 = 0.6$	1,675,098	1800	1/3	2/3	1/3
$\theta_1 = 0.5$, $\theta_2 = 0.5$	1,655,098	1900	2/3	1/3	1/3
$\theta_1 = 0.6$, $\theta_2 = 0.4$	1,655,098	1900	2/3	1/3	1/3
$\theta_1 = 0.7$, $\theta_2 = 0.3$	1,655,098	1900	2/3	1/3	1/3
$\theta_1 = 0.8$, $\theta_2 = 0.2$	1,655,098	1900	2/3	1/3	1/3
$\theta_1 = 0.9$, $\theta_2 = 0.1$	1,635,098	2000	1	0	0

Degree of feasibility (α) is fixed at 0.5 and coefficient of compensation (γ) is fixed at 0.4

5.4.3 Sensitivity analysis on γ

Table 5.3 includes the sensitivity analysis on the coefficient of compensation (γ). We fix $\theta_1 = 0.8$, $\theta_2 = 0.2$, $\alpha = 0.9$ and only vary γ to investigate it's impact on the solution. The corresponding linear membership functions are shown as follows:

$$\mu_{\zeta_1}(x_1) = \frac{2,259,891 - x_1}{190,000} \qquad \text{and} \qquad \mu_{\zeta_2}(x_2) = \frac{3100 - x_2}{600}$$

By comparing Table 5.3 and 5.2, it can be found out that the minimum satisfaction level (λ_0) is more sensitive to the coefficient of compensation (γ) than the importance of objective functions (θ_1 and θ_2). In Table 5.3, λ_0 takes value $\frac{1}{3}$ for the most cases which indicates that the model pays more attention to the objective values than the minimum satisfaction levels. In Table 5.3, λ_0 takes values such as 0, 1/3, 0.5, and 0.526 which are more diverse. Recall two objective functions are conflicting with each other: the first objective function is trying to build facilities with small fixed cost and the second objective function is aiming to build facilities with large fixed cost as they have better sewage and exhaust gas treatment systems. Therefore, solution with minimum satisfaction level greater than 0.5 is good. When $\gamma = 0.1$ or $\gamma = 0.2$, the solution is indifferent to the traditional supply chain network system because the decision makers only care about the overall system costs and ignore negative environmental impact completely. Favorable solutions in Table 5.3 will be γ between 0.7 to 0.9 since solutions are balance (μ_{W_1} and μ_{W_2} are close to each other) and minimum satisfaction level is above 0.5.

Table 5.3: Sensitivity analysis on the coefficient of compensation (γ)

	W_1 (\$)	W_2 (tons)	μ_{W_1}	μ_{W_2}	λ_0
$\gamma = 0.1$	2,069,891	3100	1	0	0
$\gamma = 0.2$	2,069,891	3100	1	0	0
$\gamma = 0.3$	2,109,891	2900	0.789	1/3	1/3
$\gamma = 0.4$	2,139,891	2800	0.632	0.5	0.5
$\gamma = 0.5$	2,139,891	2800	0.632	0.5	0.5
$\gamma = 0.6$	2,139,891	2800	0.632	0.5	0.5
$\gamma = 0.7$	2,159,891	2700	0.526	2/3	0.526
$\gamma = 0.8$	2,159,891	2700	0.526	2/3	0.526
$\gamma = 0.9$	2,159,891	2700	0.526	2/3	0.526

Degree of feasibility (α) is fixed at 0.9 and importance of objective functions (θ_1 and θ_2) are fixed at 0.8 and 0.2, respectively

5.5 Conclusions

Supply chain design is among the most critical decisions in the manufacturing production. Recently, more attention has been paid to the closed-loop supply chain systems as they provide additional profits by collecting defective/used units and remanufacturing them for consumption which recovers the value of production. In the traditional supply chain systems, flows start from suppliers, going through manufacturing plants, distribution centers and end at customers. However, closed-loop supply chain systems extend it by collecting defective/used products from customers, classifying them based on the condition, remanufacturing the recoverable units and sending recovered products back to the customers.

Closed-loop supply chain network design includes many strategic decisions such as network configuration and hence faces significant amount of uncertainties. In this paper, we consider the uncertainties in demand, return, scrap rate, manufacturing costs and environmental impacts. To copy with those uncertain parameters, a multi-objective fuzzy programming model is proposed. Two conflicting objective functions are minimization of overall system costs and minimization of negative environmental impact. We apply the solution approach proposed by Jimenez et al. to create the crisp model and then integrate different objective functions using the approach proposed by Jiménez et al. (2007) and Torabi and Hassini (2008). Sensitivity analyses have been conducted on various parameters such as the degree of feasibility (α), the importance of objective functions (θ_1, θ_2) and coefficient of compensation (γ). It can be observed that: (1) different α values will provide different linear membership functions; (2) λ_0 is insensitive to the combinations of θ_1 and θ_2; (3) By varying γ, we are able to find a balance solution.

The research is subject to a few limitations which suggest some future research directions: First, time complexity is not addressed in this paper, however, this aspect is really important for large scaled problems and hence developing valid inequalities and heuristic algorithms can be appealing. Second, an efficient approach to capture the statistical properties of uncertain parameters and convert into crisp models is desired. Last but not the least, the choice of raw materials and collection technologies play a big role in environmental impact, therefore considering uncertainties in those two components are also crucial.

CHAPTER 6. GENERAL CONCLUSIONS

A key component of a manufacturing firm is the robust design and efficient operation of its supply chain. However, supply chain design and operational planning have significant uncertainties, which complicate the planning and decision making processes. This dissertation, which consists of four manuscripts, aims to contribute to decision making under uncertainty for network design, production planning and closed-loop supply chain. The major contributions, limitations, and future works are discussed in this section.

The efficiency and cost of freight transportation are among important criteria to evaluate the overall performance of a supply chain network. In the first paper, we studied a relay network design problem. The major motivations are: First, improving long haul truck drivers' work-life balance and safety; Second, reducing deadhead cost. The contributions are: (1) We develop a novel capacitated, hub design model as a mixed-integer linear program. Unlike other relay network design systems, long distance shipments and point to point delivery are not allowed. (2) We propose a two-stage stochastic programming model with uncertain demand. Robustness of, and bottlenecks in, the deterministic system have been examined (3) Various preprocessing cuts and valid inequalities have been generated and tested. Results show that valid inequalities enhance computational performance when the size of problems are small to medium. Our study is subject to a few limitations, which suggest future research directions: strong valid inequalities should be developed to improve the computational performance of L-shaped method. Instead of using 10 scenarios to approximate the demand uncertainty, Sample Average Approximation (SAA) should be used and followed by various stability tests. Last but not least, heuristics can be used when solving large scale problem instances.

Besides freight shipment, production is another challenge in the supply chain management. Overproduction creates unnecessary inventory cost while underproduction results in not satisfying demands and thus losing customers. In the second paper, we studied a lot-sizing and scheduling problem, which included determining batch sizes and production sequences. Due to the fact that production decisions can be revised at the beginning of each time period, a multi-stage stochastic programming model has been developed. The contributions are: First, we propose a novel multi-stage stochastic programming formulation for lot-sizing and scheduling problems with demand uncertainty. Second, moment matching method followed by Fast Forward Selection approach has been utilized to generate and identify the most representative subset. The generated scenarios perfectly match the statistical properties of uncertain parameters. Third, several stability tests have been conducted in order to determine a good scenario sample size after scenario reduction process. We quantitatively measure the difference in the objective values between the two-stage and multi-stage stochastic programming approaches. Given 20 hours computational time, the objective value of multi-stage stochastic programming model is roughly 10% lower then the one in the two-stage stochastic programming model, which indicates the multi-stage stochastic programming model outperforms the two-stage stochastic programming model. However, our research has following limitations. Firstly, demand is assumed to be independent over time which can be questionable in cases where current demand heavily depends on the historical data. Secondly, multiple uncertain factors can be studied in our future works. Thirdly, we subjectively determine the scenario sample size is sufficient if the changes in the objective values is less than 5%. Last but not least, 10% drop in the objective value may not be a huge improvement in some industrial cases given 20 hours computational time.

Our first two manuscripts mainly focus on the stochastic programming approach which is a powerful tool when there is sufficient data and computation resource. However, robust optimization is more suitable tool for cases where we only have limited amount of data or the decision makers concern more about the worst case performance. In addition, scenario based stochastic programming approach requires large sample size in order to have a good representation of original distribution which can be computational expensive. In the third paper, we studied a hybrid stochastic and robust optimization approach for lot-sizing and scheduling problems. We assumed that there was not sufficient historical data for demand and hence robust optimization was applied. On the other hand, we assumed that there was enough historical data to create a reliable distribution for overtime processing cost, therefore, scenario based stochastic programming was adopted. Various sensitivity analyses have been carried out and the results show that (1) The value of budget provides a trade-off between level of constraint violation and degree of conservatism. Concretely, larger budget provides more protection to the constraints but corresponding solution becomes more conservative. On the other hand, smaller budget provides less constraint protection and corresponding solution is less conservative. (2) The objective value of the deterministic model can be either higher or lower than the objective value of the stochastic model. However, as we increase budget, the objective value of the deterministic model becomes much higher than the objective value of the hybrid model, which indicates the importance of considering uncertainties and hybrid formulation technique. One interesting future research direction is to develop strong cuts to improve the computational performance since it has been proven that lot-sizing and scheduling problems are NP-hard.

Supply chain management involves selection of suppliers, facility locations, production, movements as well as storage of goods. By integrating the network design and production planning, we studied a closed-loop supply chain problem. Because opening warehouses, distribution centers, and manufacturing plants are long term decisions, a lot of parameters in the supply chain problems have uncertainties. Fuzzy programming is preferred when there are multiple uncertain parameters since model complexity for fuzzy programming approach does not depend on the number of uncertain parameters. In the fourth paper, we proposed a fuzzy programming formulation for closed-loop supply chain problems. The main difference between traditional supply chain and closed-loop supply chain is the consideration of environmental impacts. We studied uncertain parameters such as demand, return, scrap rate, manufacturing costs and negative environmental impacts. Two objective functions minimization of overall system costs and minimization of negative environmental impacts. An aggregation function has been applied to integrate two conflicting objective functions. We observed that: (1) different α values will provide different linear membership functions; (2) λ_0 is insensitive to the combination of θ_1 and θ_2; (3) by varying γ, we are able to find a balance solution. One promising future work is to address the computational complexity of closed-loop supply chain problems.

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